Introduction to
Differential Geometry
Mth-445

Differential geometry is a mathematical discipline that uses the methods of differential and integral calculus to study problems in geometry. A fundamental contribution to the theory of surfaces was made by Gauss in 1825. His papers contain practical calculations of the curvature of the earth based purely on measurements on the surface of the planet. He considered properties of surfaces which are determined only by the geodesic distances between points on the surface independently of the particular way in which the surface is located in the ambient Euclidean space. This led to his Theorema Egregium, a fundamental result which explains why there cannot be an accurate planar map of the world.
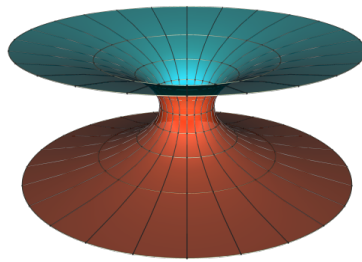

Figure 1: Catenoid

Differential Geometry allows for instance to explain and describe the beautiful surfaces created by playing with soap bubbles. They are in fact examples of "minimal surfaces" a family of geometrical object that turns out to be quite relevant in the field of the application. The above picture is the famous "catenoid" already studied by Euler.

In the applied world, Differential geometry allows to design shapes that are used in the everyday life, and is helpful when looking for a minimal path on curved surfaces. In physics, differential geometry allows to describe complicated physical systems leading to the Lagrangian formalism and general relativity. The success of these theories makes Differential geometry one of the fundamental tools in modern physics.