(1) Let \( X \) be a topological space, and define relation \( x \sim y \) if \( x \) and \( y \) lie in a common connected component. Show that \( \sim \) is an equivalence relation. Show that if \( \{ A_i \}_{i \in I} \) is a collection of connected subsets of \( X \) with \( A_i \cap A_j \neq \emptyset \) for all \( i \) and \( j \), then the union of the \( A_i \) is connected.

(2) Prove that if \( X \) has only finitely many connected components, then all the components are open.

(3) Show that a contractible space is path connected.

(4) Show that a connected open subset of \( \mathbb{R}^2 \) is path connected.

(5) Prove that \( X \) is Hausdorff if and only if the diagonal \( \Delta = \{(x, x) \mid x \in X\} \) is closed in \( X \times X \).

(6) A retract \( A \) of a space \( X \) is a subset \( A \subset X \) such that there is a continuous map \( r: X \to A \) such that \( r|_A \) is the identity on \( A \). Show that every retract \( A \) of a Hausdorff space \( X \) is closed in \( X \).

(7) Show that composition of paths satisfies the following property. If \( f_0 \cdot g_0 \simeq f_1 \cdot g_1 \), and \( g_0 \simeq g_1 \), then \( f_0 \simeq f_1 \).

(8) Let \( h \) be a path from \( x_0 \) to \( x_1 \). Show that the change of basepoint map \( \beta_h: \pi_1(X, x_1) \to \pi_1(X, x_0) \) only depends on the homotopy class of \( h \).