Homework 4
Complex Analysis, MTH 431, Spring 2014

1. Page 89: 5.3, 5.4
2. Page 98: 5.7
3. Page 102: 5.10, 5.11

4. \[ \int_{\gamma} \log z \, dz \] where \( \gamma \) is the semi-circle joining \(-i\) to \(i\) lying in right half-plane \( \text{Re}(z) \geq 0 \).

5. Let \( \gamma_1, \gamma_2, \gamma_3 \) be the following three paths from 0 to \(1 + i\):
   \[ \gamma_1(t) = t + it, 0 \leq t, \leq 1, \]
   \[ \gamma_2(t) = t + it^2, 0 \leq t, \leq 1, \]
   \[ \gamma_3(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 1 + i(t - 1) & 1 \leq t \leq 2 \end{cases} \]
   Evaluate \( \int_{\gamma_i} f(z) \, dz \), \( i = 1, 2, 3 \) for the following functions. Use any theorems we have learned to reduce computations.
   (a) \( f(z) = 2z + 1 \)
   (b) \( f(z) = 2z + 1 \)
   (c) \( f(z) = e^z \)

6. Let \( g, h : [a, b] \to \mathbb{C} \) be continuous functions and \( c = \alpha + i\beta \). Prove the following statements.
   (a) \( \int_a^b (g(t) + h(t)) \, dt = \int_a^b g(t) \, dt + \int_a^b h(t) \, dt \)
   (b) \( c \int_a^b g(t) \, dt = \int_a^b cg(t) \, dt \)

7. Let \( \gamma : [a, b] \to \mathbb{C} \) be a path and \( f_1 \) and \( f_2 \) be complex functions whose domain contains \( [\gamma] \). Use the above exercise to prove the following statements:
   (a) \( \int_\gamma (f_1(z) + f_2(z)) \, dz = \int_\gamma f_1(z) \, dz + \int_\gamma f_2(z) \, dz \)
   (b) \( c \int_\gamma f_1(z) \, dz = \int_\gamma cf_1(z) \, dz \)

8. Show that \( |\int_\gamma \frac{e^z}{z} \, dz| \leq 2\pi e \) where \( \gamma \) is the unit circle with standard parametrization.
Hand-in Problems Due: Monday March 24th 2014

1. Page 98: 5.7 a, d
2. Page 102: 5.11
3. Problem numbers 4, 5a, 7b