**PROBLEM SET**
**MTH 70200**
**REAL ANALYSIS**

**Problem 1.** (Folland 2.14) If $f \in L^+(X, \mathcal{M}, \mu)$, let $\lambda(E) = \int_E f \, d\mu$ for $E \in \mathcal{M}$. Show that $\lambda$ is a measure on $\mathcal{M}$, and for any $g \in L^+$, we have $\int g \, d\lambda = \int fg \, d\mu$. (Hint: first suppose $g$ is simple.)

**Problem 2.** (Folland 2.20) Suppose that $f_n, g_n, f, g \in L^1$ and $f_n \to f$ a.e. and $g_n \to g$ a.e., and $|f_n| \leq g_n$, and $\int g_n \to \int g$. Show that $\int f_n \to \int f$. (Hint: rework the proof of the dominated convergence theorem.)

**Problem 3.** (Folland 2.21) Suppose $f_n, f \in L^1$ and $f_n \to f$ a.e. Show that $\int |f_n - f| \to 0$ iff $\int |f_n| \to \int |f|$. (Hint: use the previous problem.)

**Problem 4.** Compute

$$\lim_{n\to\infty} \int_0^1 n^2 x^2 e^{-nx} \, dx$$

and

$$\lim_{n\to\infty} \int_0^\infty \frac{n \sin \frac{x}{n}}{x(1+x^2)} \, dx.$$

You can feel free to use elementary calculus facts. Please interpret these integrals as Lebesgue integrals with respect to Lebesgue measure.

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