

*I pledge that I have neither given nor received
unauthorized assistance during this examination.*

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a scientific calculator on this exam, but you may not use a graphing calculator.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 8 problems on 8 pages.

Question	Points	Score
1	12	
2	10	
3	10	
4	6	
5	8	
6	8	
7	10	
8	8	
Total:	72	

Good luck!

[12 points] 1. Find the derivatives of the following functions. Do **not** simplify your solutions.

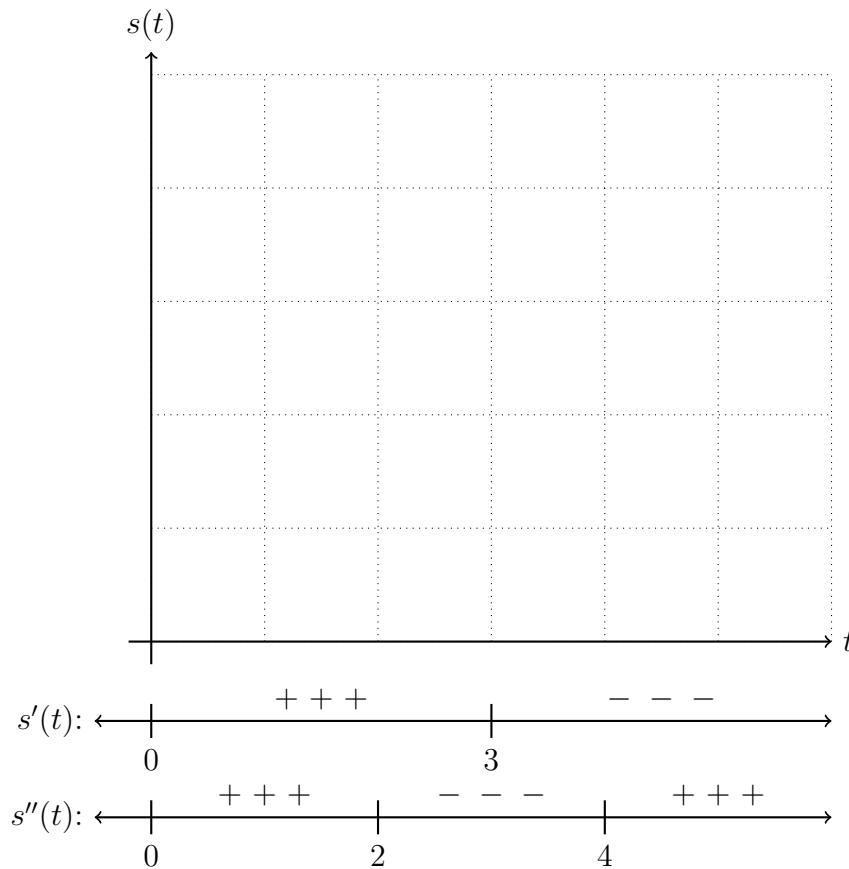
(a) $f(x) = \frac{\cos x}{x^2}$

(b) $f(x) = e^{-\sqrt{2x+1}}$

(c) $f(x) = x^3 + \frac{1}{x^2}$

- [10 points] 2. A spherical balloon is being inflated. Suppose the radius expands at a constant rate of 2 cm/s. At the moment when the radius reaches 10 cm, how quickly is the volume of the balloon growing? (Note that the volume of a sphere with radius r is $\frac{4}{3}\pi r^3$.)

- [10 points] 3. Let $s(t)$ represent the number of subscribers of a streaming service at time t years (with $t = 0$ representing 2015, when the service started). The sign diagrams of $s'(t)$ and $s''(t)$ are given below.



- (a) At time $t = 1$, which of the following is true about the streaming service:
- It's gaining subscribers, and the rate of gain is increasing.
 - It's gaining subscribers, but the rate of gain is decreasing.
 - It's losing subscribers, and the rate of loss is increasing.
 - It's losing subscribers, but the rate of loss is decreasing.
- (b) At time $t = 5$, which of the following is true about the streaming service:
- It's gaining subscribers, and the rate of gain is increasing.
 - It's gaining subscribers, but the rate of gain is decreasing.
 - It's losing subscribers, and the rate of loss is increasing.
 - It's losing subscribers, but the rate of loss is decreasing.

- (c) At what times does the number of subscribers achieve a local minimum? If the answer is never, say so.
- (d) At what times does the number of subscribers achieve a local maximum? If the answer is never, say so.
- (e) Give the t -coordinates of all inflection points of $s(t)$, or state that there are none.
- (f) Sketch the graph of $s(t)$ on the axes above the sign diagram. Assume that at time $t = 0$, the service has 0 users.

[6 points] 4. Compute

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x}.$$

[8 points] 5. Let $f(x) = \ln(2x + 1)$. Give an estimate of $f(.1)$ using linearization.

- [8 points] 6. Let $g(x) = x^3 - 3x + 2$. Find the absolute maximum and minimum of $g(x)$ on the interval $[0, 3]$.

7. Let $f(x) = \frac{(x-1)^2}{x^2}$. The first two derivatives of this function can be computed to be

$$f'(x) = \frac{2(x-1)}{x^3},$$

$$f''(x) = -\frac{2(2x-3)}{x^4}.$$

[3 points] (a) Give the locations of any vertical asymptotes of the function, or state that it doesn't have any.

[3 points] (b) List all critical values of this function.

[4 points] (c) Give the intervals where this function is increasing and the intervals where it is decreasing.

- [8 points] 8. Consider the curve defined by equation $x^2y^3 + x^2 = 2$. Find the equation for the tangent line to the curve at $(1, 1)$.