

1. (10 points) Show that the following table given in Baxter/Rennie, p.2, is correct.

Actual probability	25%	75%	
Bets placed	\$5000	\$10000	
1. Quoted odds	13-5 against	15-4 on	
Implied probability	28%	79%	Total = 107%
Profit if horse wins	-\$3000	\$2333	Expected profit = \$1000
2. Quoted odds	9-5 against	5-2 on	
Implied probability	36%	71%	Total = 107%
Profit if horse wins	\$1000	\$1000	Expected profit = \$1000

2. (10 points) Show that the variance of the Binomial distribution is given by $np(1 - p)$.
3. (10 points) Show that the following formula holds for a random variable X with a Gaussian distribution:

$$\mathbb{E}(\exp(\theta X)) = \exp\left(\theta\mu + \frac{1}{2}\theta^2\sigma^2\right)$$

1. (10 points) (A real 'classic') Welcome to your interview! Sit in this chair. Now we are going to play "Russian roulette". I have a revolver with six empty chambers. Watch me as I load the weapon with two contiguous rounds (i.e., two bullets side-by-side in the cylindrical barrel). Watch me as I spin the barrel. I am putting the gun against your head. Close your eyes while I pull the trigger. Click! This is your lucky day: you are still alive! Our game differs from regular Russian roulette because I am not going to add any bullets to the barrel before we continue, and I am not going to give you the gun. My question to you: I am going to shoot at you once more before we talk about your resume. Do you want me to spin the barrel once more, or should I just shoot?
2. (10 points) Biased wheel of fortune: Assume that the probability for an angle ϕ is $P(\phi) = \lambda\phi^2$. I pay \$1000 times the angle.
 - (a) What is λ ?
 - (b) Find the expectation and the variance of the game.
3. (10 points) A stock price is currently \$50. It is known that at the end of 2 months it will be either \$53 or \$48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a 2-month European call option with a strike price of \$49?
4. (Extra credit: 10 points) A coin-making machine produces pennies. Each penny is manufactured to have a probability P of turning up heads. However, the machine draws P randomly from the uniform distribution on $[0, 1]$, so P can differ for each coin produced. A coin pops out of the machine. You flip it once, and it comes up heads. Given this information, what is the (conditional) distribution function $F_{P|H}(p)$ for the probability of a head for that coin (where "H" denotes conditioning on the heads)? Hint: You are looking for $F_{P|H}$. Let "A" denote the event that $P \leq p$, and "H" denotes the event that you get a head. Now use Bayes' Theorem:

$$F_{P|H} = P(P \leq p|H) = P(A|H) = \frac{P(A \cap H)}{P(H)}$$

and try to find a way to compute those probabilities using the uniform distribution.

1. (10 points) A stock price is currently \$50. It is known that at the end of 6 months it will be either \$60 or \$42. The risk-free rate of interest with continuous compounding is 12% per annum. Calculate the value of a 6-month European call option on the stock with an exercise price of \$48.
2. (10 points) A stock price is currently \$25. It is known that at the end of 2 months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of 2 months. What is the value of a derivative that pays off S_T^2 at this time?
3. (10 points) (Baxter/Rennie Exercise 2.1) Show that a forward contract, struck at k , can be thought of as the payoff f , where $f(2) = s_2 - k$ and $f(3) = s_3 - k$. Now verify, using the formula for V , that the correct strike price is indeed $s_1 \exp(r\delta t)$.

1. (10 points) (See Baxter/Rennie, ex. 2.2) The stock process discussed in class starts at time 0 at 100 and then can go up and down 20 for each time step. We consider three time steps, the risk-free interest rate is zero. Repeat the calculations for this stock process for a digital contract that pays off 100 if the stock ends higher than it started.
2. (10 points) Consider a 2-year European put (right to sell) with a strike price of \$52 on a stock whose current price is \$50. We suppose that there are two time steps of 1 year, and in each time step the stock price either moves up by 20% or moves down by 20%. We also suppose that the risk-free interest rate is 5%.
 - (a) Sketch a tree of the stock process.
 - (b) What is the value of the option after 2 years?
 - (c) Find the option value at all nodes of the tree.
3. (10 points, extra credit) Find the option price at all nodes of the above tree (same stock, same interest rate) for an American put option (American means that the option can be exercised at any time step).