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ASYMPTOTICS OF CERTAIN RANDOM FIELDS ON A CIRCLE

R.S. ELLIS* – J.S. ROSEN**

ABSTRACT

We study the asymptotic behavior of sums S_n of random variables defined for certain mean-field type ferromagnetic systems on a circle. The probability distribution of S_n may be expressed in terms of a certain measure on a space $\mathscr{Y}$ of continuous functions. Suitably scaled and centered versions of this measure have limits, in terms of which the asymptotic behavior of S_n is determined. These function space limits depend crucially upon the minimum points of a nonlinear functional on $\mathscr{Y}$ related to the specific free energy of the ferromagnetic system. The study of these limits is related to work by M. Donsker, S. Varadhan, and other authors. The proofs of the function space limit results are lengthy and will appear elsewhere.

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1. INTRODUCTION

For each $n \in \{1, 2, \ldots\}$ we define a mean-field-type ferromagnetic system on the sites $\left\{\frac{j}{n}; j = 1, \ldots, n\right\}$ of a circle of circumference one. Let $\{X_j^{(n)}; j = 1, \ldots, n\}$ denote the random variables which measure the strengths of the magnetic moments at the sites $\left\{\frac{j}{n}\right\}$. The joint distribution of the $\{X_j^{(n)}\}$ – i.e., the Gibbs measure of the system – is defined to be

$$d\Gamma_n(X_1, \ldots, X_n) :=$$

$$:= \frac{1}{Z_n} \exp\Big[\frac{1}{2n} \sum_{1 \leq j, l \leq n} J\left(\frac{j}{n} - \frac{l}{n}\right) X_j X_l + \sum_{j=1}^{n} H\left(\frac{j}{n}\right) X_j\Big] \prod_{j=1}^{n} d\rho(X_j), \tag{1.1}$$

where

$$Z_n := \int_{R^n} \exp\Big[\frac{1}{2n} \sum_{1 \leq j \leq l \leq n} J\left(\frac{j}{n} - \frac{l}{n}\right) X_j X_l + \sum_{j=1}^{n} H\left(\frac{j}{n}\right) X_j\Big] \prod_{j=1}^{n} d\rho(X_j). \tag{1.2}$$

We have set $\beta := \frac{1}{kT}$ equal to one (k is Boltzmann's constant, T the absolute temperature). Among all Gibbs measures on the sites $\left\{\frac{j}{n}\right\}$ with isotropic pair interaction potentials J satisfying out hypotheses, $d\Gamma_n$ is essentially the only one which exhibits interesting limiting behavior. This is explained in Remark 4.2.

Let $\mathscr{Y}$ denote the space of all real-valued continuous functions on R which are periodic of period one. The function $J(t)$ in (1.1)-(1.2) is an even, positive, suitably smooth element of $\mathscr{Y}$ which also satisfies the technical hypotheses listed at the start of Section 3. The most important of these is that J be positive definite (i.e., each Fourier coefficient positive). We normalize J so that

$$\int_0^1 J(t)\, dt = 1. \tag{1.3}$$