MTH 330 Laplace Transform REVIEW SHEET II Due: March 6, 2020

1. Solve the the following initial value problem:

$$y'' + 12y' + 40y = \delta(t);\ \ y(0) = 0,\ y'(0) = 0$$

2. Solve the the following initial value problem:

$$y'' + 144y = \delta(t);\ \ y(0) = 0,\ y'(0) = 0$$

3. What is a convolution of two functions? What is the Laplace transform of a convolution product? (ie What is $\mathcal{L}[f * g]$)?

4. Compute the following convolution products:

   (a) $e^{-3t} * e^{2t}$

   (b) $H(t) * \cos 3t$

   (c) $t * t^2$

5. Find the inverse Laplace transform of the following:

   (a) $T(s) = \left(\frac{1}{s+3}\right)\left(\frac{1}{s-2}\right)$

   (b) $T(s) = \left(\frac{1}{s}\right)\left(\frac{s}{s^2+9}\right)$

   What is the connection to the results of Question 1?

6. Use the convolution theorem to find the inverse transforms of the following in terms of an integral. Do the integration if you can.

   (a)

$$T(s) = \left(\frac{4}{s+4}\right)\left(\frac{1}{s-6}\right)$$

   (b)

$$T(s) = \frac{3}{s(s^2+16)}$$

7. Use the convolution theorem to write the solution to the following IVP in terms of an integral FOR ANY $f(t)$

$$y'' + 6y' + 5y = f(t);\ \ y(0) = 0,\ y'(0) = 0$$

   Use this general solution to find $y(t)$ when $f(t) = e^{-2t}$.