MTH/BIO 415 - Fall 2013

Homework Assignment #2: Nonlinear Difference Equations

Due: Monday, September 16

1. Indicate whether each of the following difference equations is linear or nonlinear. If linear, solve the equation and state whether the solution decays or grows with increasing n. If nonlinear, find ALL equilibrium solutions.

   (a) $x_{n+1} = rx_n(1 - x_n^2)$

   (b) $x_{n+1} = 3x_n - x_{n-1}$

   (c) $x_{n+1} = \frac{x_n}{1+x_n}$

   (d) $x_{n+1} = rx_n(\alpha - x_n^2)$, α = constant

2. Determine the equilibrium values and thier stability for the following nonlinear difference equation:

$$x_{n+1} = -\frac{x_n^2}{2}(1 - x_n)$$

3. A frequently encountered model for fish populations is based on an empirical relation called the *Ricker Equation*

$$N_{n+1} = \alpha N_n e^{-\beta N_n} \quad .$$

   Here α represents the maximal growth rate of the organism and β is the inhibition of growth caused by overpopulation.

   (a) Use MatLab to sketch a graph of the population dependent growth rate. Determine how this graph changes with changes to α and β.

   (b) Determine all the steady state (equilibrium) populations:

   (c) Determine the conditions for each steady state to be stable.

   (d) Write a matlab script like `logmap.m` to simulate the Ricker Equation. Demonstrate the results determined above with Matlab. (In other words, use the program to show that the steady state is stable/unstable depending on the size of $|1 - \ln \alpha|$.)