Homework Assignment #7: Predator Prey - Fitting Data
Do Hares Eat Lynx??
Due: Wednesday, November 27 (in my mailbox?)

1. **Two Predator Prey Models:** In class we have discussed classic ODE models for the populations of interacting species. Two classics are:

   Model I:

$$\begin{aligned} \frac{dH}{dt} &= a_1 H - a_2 HL - a_3 H^2 \\ \frac{dL}{dt} &= -b_1 L + b_2 HL \end{aligned}$$

   Model II:

$$\begin{aligned} \frac{dH}{dt} &= a_1 H - a_2 \frac{HL}{1 + k_1 H} \\ \frac{dL}{dt} &= -b_1 L + b_2 \frac{HL}{1 + k_1 H} \end{aligned}$$

   a) For each model, carefully explain what each term is attempting to represent. Explain what the differences are between the two models.

   b) What are the dimensions of each of the parameters in each of the models. Explain, clearly, what these parameters represent biologically.

   c) For each of the models, write down NON-DIMENSIONAL versions. What are the important groups of parameters? How many non-dimensional parameter groups in each model?

   d) For each of the non-dimensional models, carefully sketch the null-clines and find all equilibrium points in the phase plane. Do this for different values of the non-dimensional parameter groups.

   e) Use `ODE45` or `PPLANE8` to investigate solutions to the each model. Discuss the stability of the equilibrium points.

2. **Fitting Parameters to Data** The table below gives historical data on the population of lynx and hare.

Year	Hare (×1000)	Lynx (×1000)	Year	Hare (×1000)	Lynx (×1000)
1900	30	4	1916	11.2	29.7
1901	47.2	6.1	1917	7.6	15.8
1902	70.2	9.8	1918	14.6	9.7
1903	77.4	35.2	1919	16.2	10.1
1904	36.3	59.4	1920	24.7	8.6
1905	20.6	41.7	1921	33	18.8
1906	18.1	19	1922	37.1	25.6
1907	21.4	13	1923	41.2	34.2
1908	22	8.3	1924	35.7	41.9
1909	25.4	9.1	1925	14.9	47
1910	27.1	7.4	1926	4.5	53
1911	40.3	8	1927	0.9	42.7
1912	57	12.3	1928	0.9	31.6
1913	76.6	19.5	1929	1.8	20.5
1914	52.3	45.7	1930	2.7	10.3
1915	19.5	51.1			

Data from the Hudson Bay Trading Company on Hare and Lynx Populations

a) Somehow, reformat the data so it can be input into Matlab. Then plot the data on the hare-lynx phase plane. Be sure to label the initial, intermediate and final points somehow so you have an clear graphic of how the population changes in time. Also make a separate plot showing $H(t)$ and $L(t)$ versus time.

c) Experiment with parameters and initial conditions in Model I to get solutions that show some level of agreement with the data. In other words, plot solutions of Model I against the data on the H-L phase plane.

d) We now want to seriously test our models against this data. Use the R code you already investigated for the SIR disease models to find the best fit for the parameters and initial conditions in both models. Start with the simpler, Model I, and find a_1, a_2, a_3, b_1, b_2 and initial values that best fit the data. Use the initial guesses for the parameters you found in part (c).

e) Use the best-fit values you found from R in part (d) in Model-I and compare the output of the model to the data. Comment on the results. Does the model, using the best fit parameters, capture the hare-lynx population dynamics given in the data?

f) Repeat the analysis for Model-II. From all this, decide which is the 'better' model. Why? Be explicit!