MTH/BIO 415 MIDTERM REVIEW SHEET

MIDTERM: Wednesday, March 29, 2006

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1. Find the eigenvalues and eigenvectors of the following matrix. (Do it by hand!)

$$A = \begin{pmatrix} 4 & -2 \\ 3 & -2 \end{pmatrix}$$

2. Determine whether the following set of difference equations has solutions which tend to infinity or to zero

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 2 & 4\frac{1}{4} \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

3. Consider the following discrete time population model for a single species

$$N_{t+1} = N_t \exp\left[r(1 - N_t/K)\right]$$

   a) Describe in words what the parameters r and K represent biologically.

   b) Find all steady state solutions to the model.

   c) Determine the stability of each steady state.

   d) Given $r = 0.5$ and $K = 2,000$, what would you expect the model to predict for the population. Sketch a graph of N versus t.

4. Consider the following simple matrix model for a certain species of turtle grouped into 3 yearly age classes.

$$\mathbf{X}_{n+1} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{pmatrix} = \begin{pmatrix} 0.90 & 0.35 & 0.00 \\ 0.00 & 0.00 & 0.25 \\ 0.30 & 0.00 & 0.00 \end{pmatrix} \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix} = \mathbf{A}\mathbf{X}_n$$

   (a) Explain, in words, what you think the model is trying to represent. What are the x, y, z variables representing. What does each term in the matrix represent?

   (b) A savvy student takes the matrix and computes its eigenvalues and eigenvectors using MatLab. She gets the following results:

```
>> [v,d] = eig(A)

v =
   0.9554            -0.0209 + 0.3499i  -0.0209 - 0.3499i
   0.0955            -0.1195 - 0.7893i  -0.1195 + 0.7893i
   0.2796             0.4892 - 0.0222i   0.4892 + 0.0222i


d =
   1.0250                  0                  0
        0            -0.0225 + 0.2135i        0
        0                  0            -0.0225 - 0.2135i
```

Use these results to answer the following questions:

i. What does the model predict will happen to the turtle population as time goes on? Why?

ii. After a long time (say 100 years), what will be the distribution of turtles among age classes? (ie: What percentage of turtles will be in the middle (y) age class? What percentage will be in the old (x) age class?)

(c) This same, savvy student wonders how sensitive the matrix is to changes in parameters. She gets hold of some sensitivity code and computes:

```
>> [S,E] = sense(A)
S =

    0.9193    0.0919    0.2691
    0.4036    0.0403    0.1181
    0.1378    0.0138    0.0403

E =
    0.8790    0.0403         0
         0         0    0.0403
    0.0403         0         0
```

What does this all mean as far as the turtles go? Given the model and the sensitivity results, what is the most important biological concern for the continued health of the turtle species? EXPLAIN fully.

5. Beddington and May (1982) have proposed the following model to study the interactions between baleen whales (y) and their main food source, krill x, in the Southern ocean:

$$\begin{aligned} \dot{x} &= rx\left(1-\frac{x}{K}\right) - axy \\ \dot{y} &= sy\left(1-\frac{y}{bx}\right) . \end{aligned}$$

a) Explain, in words, what each term in the model is trying to describe. What do the parameters r, s, a, b and K represent?

b) Find all the *null-clines* of the model.

c) Find the fixed points.

d) (Probably EXTRA CREDITish) Determine the stability of the fixed points graphically - Sketch a few solutions in the phase plane.

6. Write down a simple continuous model for a single species population that grows *logistically*. Describe the dynamics of the model - fixed points, stability, direction-field graphical analysis.