Positive Solutions for Nonlinear Elliptic Systems with Combined Nonlinear Effects

Ratnasingham Shivaji

Department of Mathematics and Statistics
Center for Computational Sciences
Mississippi State University

Abstract

We study the existence and multiplicity of positive solutions to \( n \times n \) systems of the form

\[
-\Delta u_i = \lambda f_i(u_{i+1}) \quad \text{in } \Omega \\
\vdots \\
-\Delta u_{n-1} = \lambda f_{n-1}(u_n) \quad \text{in } \Omega \\
-\Delta u_n = \lambda f_n(u_1) \quad \text{in } \Omega \\
u_1 = u_2 = \cdots = u_n = 0 \quad \text{on } \partial \Omega
\]

Here \( \Delta \) is the Laplacian operator, \( \lambda \) is a non-negative parameter, \( \Omega \) is a bounded domain in \( \mathbb{R}^N \) with smooth boundary \( \partial \Omega \) and \( f_i \in C^1([0, \infty)) \), \( i \in \{1, 2, \ldots, n\} \) belongs to a class of strictly increasing functions that have a combined sublinear effect at \( \infty \). We establish results for positone systems \( f_i(0) \geq 0, \ i \in \{1, \ldots, l - 1, l + 1, \ldots, n\} \) and \( f_l(0) > 0 \) for some \( l \in \{1, \ldots, n\} \). We establish our results by the method of sub and supersolutions. We also discuss our results in the case when one of the nonlinearities, say \( f_k \), is given by \( f_k(z) = e^{\alpha z^2} ; \ \alpha > 0 \) which arises in the theory of combustion.