Math 232 Calculus 2 Fall 17 Midterm 1a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int \frac{e^{-2x}}{e^{-2x}+2}\,dx$.

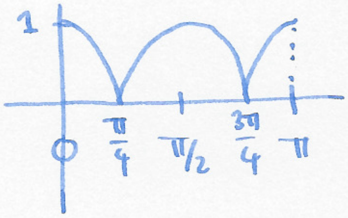
$u = e^{-2x}$

$\frac{du}{dx} = -2e^{-2x}$

$\int \frac{e^{-2x}}{u+2} \cdot \frac{dx}{du}\,du = \int \frac{e^{-2x}}{u+2}\,\frac{1}{-2e^{-2x}}\,du$

$= -\frac{1}{2}\int \frac{1}{u+2}\,du = -\frac{1}{2}\ln|u+2| + C$

(2) (10 points) Find $\displaystyle\int_0^{\pi} |\cos(2x)|\ dx$. Draw a picture of the region.



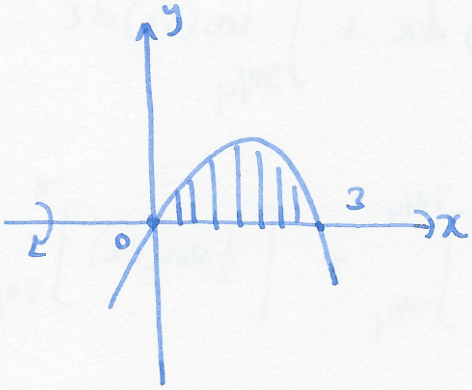
$$\int_0^{\pi/4} \cos(2x)\,dx + \int_{\pi/4}^{3\pi/4} -\cos(2x)\,dx + \int_{3\pi/4}^{\pi} \cos(2x)\,dx$$

$$= \left[\tfrac{1}{2}\sin(2x)\right]_0^{\pi/4} - \left[\tfrac{1}{2}\sin(2x)\right]_{\pi/4}^{3\pi/4} + \left[\tfrac{1}{2}\sin(2x)\right]_{3\pi/4}^{\pi}.$$

$$= \tfrac{1}{2} - 0 \quad - \left(-\tfrac{1}{2}\right) + \tfrac{1}{2} \quad + 0 - \tfrac{1}{2}(-1)$$

$$= 2$$

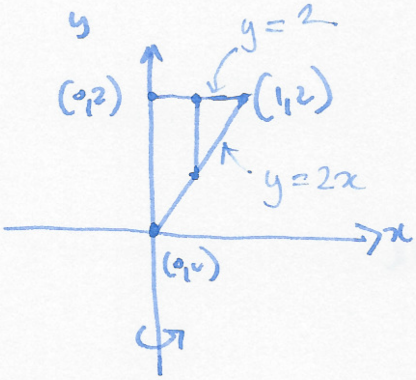
(3) (10 points) Draw a picture of the region bounded by the curve $y = 3x - x^2$, for $y \geq 0$. Write down an integral to give you the volume of revolution of this region about the x-axis. DO NOT EVALUATE THIS INTEGRAL.



discs : $V = \int_0^3 \pi r^2 \, dx$

$$= \int_0^3 \pi \left(3x - x^2\right)^2 dx$$

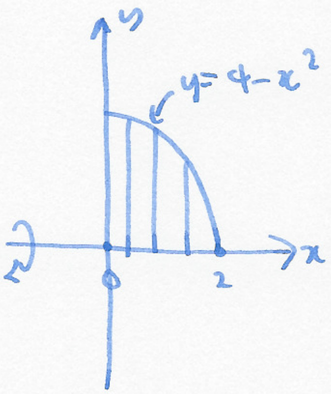
(4) (10 points) Use shells to write down an integral for the volume of the cone formed by rotating the triangle with vertices $(0,0)$, $(0,2)$ and $(1,2)$ about the y-axis. DO NOT EVALUATE THIS INTEGRAL.



shells : $V = \int_0^1 2\pi r h \, dx$

$= \int_0^1 2\pi x (2-2x) \, dx$

(5) (10 points) Consider the subset of the plane bounded by $y = 4 - x^2$ in the first quadrant (i.e. $x \geq 0$ and $y \geq 0$). Find the volume of revolution of the 3-dimensional shape formed by rotating this region around the x-axis.



discs: $V = \int_0^2 \pi r^2 \, dx$

$= \int_0^2 \pi (4-x^2)^2 \, dx$

$= \pi \int_0^2 16 - 8x^2 + x^4 \, dx = \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2$

$= \pi \left(32 - \frac{16}{3} + \frac{64}{5} \right) \approx \cancel{124}\ 53.6$

$\cancel{\frac{\pi\, 892}{16}} = \frac{256\pi}{15}$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(7) (10 points) Find $\int \underbrace{e^{-3x}}_{u} \underbrace{\cos(x)}_{v'}\, dx$.

$u = e^{-3x}$ $\quad u' = -3e^{-3x}$

$v' = \cos x$ $\quad v = \sin x$

$$\int e^{-3x}\cos(x)\,dx = e^{-3x}\sin x - \int -3e^{-3x}\sin x\,dx$$

$$= e^{-3x}\sin x + 3\int \underbrace{e^{-3x}}_{u}\underbrace{\sin x}_{v'}\,dx$$

$u = e^{-3x}$ $\quad u' = -3e^{-3x}$

$v' = \sin x$ $\quad v = -\cos x$

$$\int e^{-3x}\cos(x)\,dx = e^{-3x}\sin x \;\cancel{+}- 3e^{-3x}\cos x \;\cancel{+}- 3\cdot\int 3e^{-3x}\cos x\,dx$$

$$\int e^{-3x}\cos(x)\,dx = +\frac{1}{10}\left(e^{-3x}\sin x - 3e^{-3x}\cos x\right) + c.$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(6) (10 points) Find $\int \sqrt[3]{x} \ln(3x)\, dx$.

$$\int \underbrace{x^{1/3}}_{v'} \underbrace{\ln(3x)}_{u} dx = \frac{3x^{4/3}}{4} \ln(3x) - \int \frac{3x^{4/3}}{4} \cdot \frac{1}{x}\, dx$$

$u = \ln(3x)$ $\quad u' = \frac{1}{3x} \cdot 3 = \frac{1}{x}$

$v' = x^{1/3}$ $\quad v = \frac{3x^{4/3}}{4}$

$$= \frac{3}{4} x^{4/3} \ln(3x) - \int \frac{3}{4} x^{1/3}\, dx$$

$$= \frac{3}{4} x^{4/3} \ln(3x) - \frac{9}{16} x^{4/3} + C.$$

(8) Find $\int_0^{\pi/2} \sin^3 x \, dx$. $= \displaystyle\int_0^{\pi/2} \sin x \left(1-\cos^2 x\right) dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$$\int_1^0 \sin x \left(1-u^2\right) \frac{dx}{du} \, du$$

$$= \int_1^0 \sin x \left(1-u^2\right) \frac{1}{-\sin x} \, du = \int_0^1 1-u^2 \, du = \left[u - \frac{1}{3}u^3\right]_0^1$$

$$= 1 - \frac{1}{3} = \frac{2}{3}.$$

$= \cancel{\cos x - \frac{1}{3}\cos^3 x + C}$

(9) Find $\int \sin(5x)\sin(3x)\,dx$.

$$\left.\begin{aligned}\cos(A+B) &= \cos A\cos B - \sin A\sin B\\ \cos(A-B) &= \cos A\cos B + \sin A\sin B\end{aligned}\right\} \quad \cos(A-B) - \cos(A+B) = 2\sin A\sin B.$$

$$= \int \frac{1}{2}\left(\cos(2x) - \cos(8x)\right)dx = \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + C$$

(10) Find $\int \frac{1}{9+x^2}\,dx.$ $= \frac{1}{9}\int \frac{1}{1+x^2/9}\,dx$

$u = \frac{x}{3}$

$\frac{du}{dx} = \frac{1}{3}$

$\frac{1}{9}\int \frac{1}{1+u^2}\frac{dx}{du}\,du = \frac{1}{9}\int \frac{1}{1+u^2}\cdot 3\cdot du$

$= \frac{1}{3}\tan^{-1}(u) + c = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$