

Sample midterm 1 solutions

①

Q1 $\int -2x^2 \cos(-2x^3) dx$ $u = -2x^3$
 $\frac{du}{dx} = -6x^2$

$$\int -2x^2 \cos(u) \frac{dx}{du} du = \int -2x^2 \cos(u) \frac{1}{-6x^2} du = \int \frac{1}{3} \cos(u) du$$

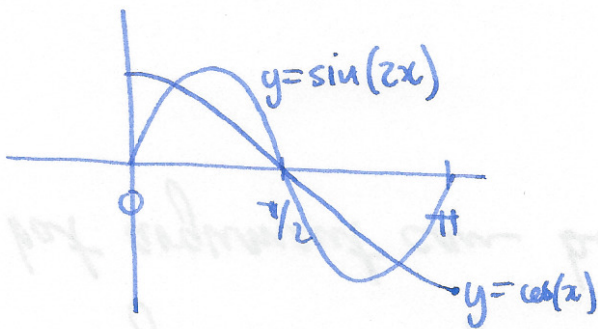
$$= \frac{1}{3} \sin(u) + c = \frac{1}{3} \sin(-2x^3) + c$$

Q2 $\int 3x^3 (1-x^4)^{1/5} dx$ $u = 1-x^4$
 $\frac{du}{dx} = -4x^3$

$$\int 3x^3 u^{1/5} \frac{dx}{du} du = \int 3x^3 u^{1/5} \frac{1}{-4x^3} du = \int -\frac{3}{4} u^{1/5} du = -\frac{3}{4} \frac{5}{6} u^{6/5} + c$$

$$= -\frac{5}{12} (1-x^4)^{6/5} + c$$

Q3



solve: $\sin(2x) = \cos(x)$

$$2\sin x \cos x = \cos x$$

$$\cos x (2\sin x - 1) = 0$$

$$\cos x = 0 \Leftrightarrow x = \pi/2 \pm n\pi \quad x = \pi/2$$

$$\sin x = \frac{1}{2} \Leftrightarrow \pm \frac{\pi}{6} \pm n\pi \quad x = \frac{5\pi}{6}$$

$$\int_{\pi/2}^{5\pi/6} \cos x - \sin 2x dx + \int_{5\pi/6}^{\pi} \sin 2x - \cos x dx$$

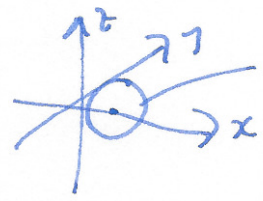
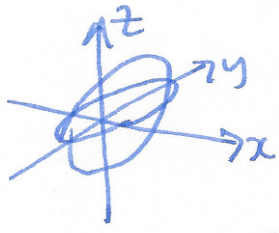
$$= \left[\sin x + \frac{1}{2} \cos 2x \right]_{\pi/2}^{5\pi/6} + \left[-\frac{1}{2} \cos 2x - \sin x \right]_{5\pi/6}^{\pi}$$

$$= \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} - \left(1 + \frac{1}{2}(-1) \right) + \left(-\frac{1}{2} \cdot 1 - 0 \right) - \left(-\frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \right)$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

②

Q4



$y^2 + z^2 = 1 - 16x^2$
 circle of radius $r^2 = 1 - 16x^2$
 a) area $\pi r^2 = \pi(1 - 16x^2)$
 of vertical cross section.

$$b) v = \int A(x) dx = \int_{-1/4}^{1/4} \pi(1 - 16x^2) dx = \pi \left[x - \frac{16}{3} x^3 \right]_{-1/4}^{1/4}$$

$$= \pi \left(\left(\frac{1}{4} - \frac{16}{3} \cdot \frac{1}{64} \right) - \left(-\frac{1}{4} + \frac{16}{3} \cdot \frac{1}{64} \right) \right)$$

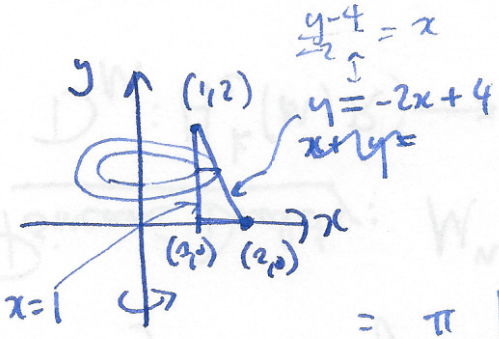
$$= \pi \left(\frac{1}{2} - \frac{1}{12} \right) = \frac{5\pi}{12}$$

Q5

$$\frac{1}{4} \int_{-2}^2 e^{-x/4} dx = \frac{1}{4} \left[-4e^{-x/4} \right]_{-2}^2 = \frac{1}{4} \left(-4e^{-1/2} + 4e^{1/2} \right)$$

$$= e^{1/2} - e^{-1/2}$$

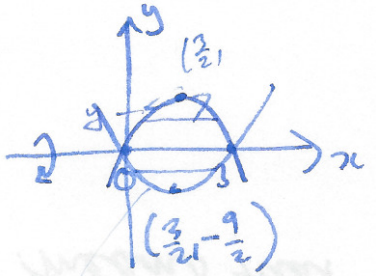
Q6



$$\pi \int_0^2 \left(2 - \frac{1}{2}y \right)^2 - 1^2 dy = \pi \int_0^2 \left(3 - 2y + \frac{1}{4}y^2 \right) dy$$

$$= \pi \left[3y - y^2 + \frac{1}{12}y^3 \right]_0^2 = \pi \left(6 - 4 + \frac{8}{12} \right) = \pi \frac{8}{3}$$

Q7



$y = 2x^2 - 6x$
 $2x^2 - 6x - y = 0$
 $x = \frac{6 \pm \sqrt{36 + 8y}}{4}$

$$2\pi \int_{-9/2}^0 y \cdot \left(\frac{6 + \sqrt{36 + 8y}}{4} - \frac{6 - \sqrt{36 + 8y}}{4} \right) dy$$

$$= 2\pi \int_{-9/2}^0 y \cdot \frac{\sqrt{36 + 8y}}{2} dy = 2\pi \int_{-9/2}^0 y \sqrt{9 + 2y} dy$$

$$u = 9 + 2y$$

$$\frac{du}{dy} = +2$$

$$2\pi \int_{180}^9 (u-9)^{1/2} \sqrt{u} \cdot \frac{1}{+2} du = 2\pi \int_{180}^9 u^{3/2} - 9u^{1/2} du$$

$$= \frac{2\pi}{4} \left[\frac{2u^{5/2}}{5} - 9 \cdot \frac{2}{3} u^{3/2} \right]_{180}^9 = \frac{2\pi}{4} \left[\left(\frac{2}{5} \cdot 9^{5/2} - 6 \cdot 9^{3/2} \right) - \left(\frac{2 \cdot 18^{5/2}}{5} - 6 \cdot 18^{3/2} \right) \right]$$

$$= 32 \cdot 4\pi$$

Q8 $\int x \ln(x+1) dx$ $\int u'v dx = uv - \int uv'dx$

$$u' = x \quad u = \frac{1}{2}x^2$$

$$v = \ln(x+1) \quad v' = \frac{1}{x+1}$$

$$= \frac{1}{2}x^2 \ln(x+1) - \int \frac{1}{2}x^2 \cdot \frac{1}{x+1} dx$$

$$z = x+1$$

$$\frac{dz}{dx} = 1$$

$$= \frac{1}{2}x^2 \ln(x+1) - \frac{1}{2} \int (z-1)^2 \frac{1}{z} dz = \frac{1}{2}x^2 \ln(x+1) - \frac{1}{2} \int z - 2 + \frac{1}{z} dz$$

$$= \frac{1}{2}x^2 \ln(x+1) - \frac{1}{4}z^2 + z - \frac{1}{2} \ln(z) + c = \frac{1}{2}x^2 \ln(x+1) - \frac{1}{4}(x+1)^2 + (x+1) - \frac{1}{2} \ln(x+1) + c$$

Q9 $\int \underbrace{e^{-3x}}_{u'} \underbrace{\cos(2x)}_v dx = -\frac{1}{3} e^{-3x} \cos(2x) - \int -\frac{1}{3} e^{-3x} \cdot -2 \sin(2x) dx$

$$u' = e^{-3x} \quad u = -\frac{1}{3} e^{-3x}$$

$$v = \cos(2x) \quad v' = -2 \sin(2x)$$

$$\int e^{-3x} \cos(2x) dx = -\frac{1}{3} e^{-3x} \cos(2x) - \int \frac{2}{3} e^{-3x} \sin(2x) dx$$

$$u' = \frac{2}{3} e^{-3x} \quad u = -\frac{2}{9} e^{-3x}$$

$$v = \sin(2x) \quad v' = 2 \cos(2x)$$

$$\int e^{-3x} \cos(2x) dx = -\frac{1}{3} e^{-3x} \cos(2x) - \frac{2}{9} e^{-3x} \sin(2x) + \int -\frac{2}{9} e^{-3x} \cdot 2 \cos(2x) dx$$

$$\frac{13}{9} \int e^{-3x} \cos(2x) dx = -\frac{1}{3} e^{-3x} \cos(2x) + \frac{2}{9} e^{-3x} \sin(2x) + C.$$

$$\int e^{-3x} \cos(2x) dx = -\frac{3}{13} e^{-3x} \cos(2x) + \frac{2}{13} e^{-3x} \sin(2x) + C.$$

Q10 $\int \frac{x e^x \sin x}{v \cdot u'} dx$ $\int u' v dx = uv - \int uv' dx.$

$u' = e^x \sin x$ $u = ?$
 $v = x$ $v' = 1$

$\int \frac{e^x \sin x}{u' \cdot v} dx = e^x \sin x - \int \frac{e^x \cos x}{u' \cdot v} dx$
 $u' = e^x$ $u = e^x$
 $v = \sin x$ $v' = \cos x$

$u' = e^x$ $u = e^x$
 $v = \cos x$ $v' = -\sin x$

$$\int e^x \sin x dx = e^x \sin x - e^x \cos x + \int e^x \sin x dx$$

$$\int e^x \sin x dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x$$

$$\int x e^x \sin x dx = x \frac{1}{2} e^x (\sin x - \cos x) - \frac{1}{2} \int e^x \sin x - e^x \cos x dx.$$

done this already

$$\int \frac{e^x \cos x}{u' \cdot v} dx = + \frac{e^x \cos x}{u \cdot v} + \int \frac{e^x \sin x}{u' \cdot v} dx$$

$$= e^x \cos x + e^x \sin x - \int e^x \cos x dx$$

$$\int x e^x \sin x dx = \frac{1}{2} x e^x (\sin x - \cos x) - \frac{1}{2} \left(\frac{1}{2} (e^x (\sin x - \cos x)) + \frac{1}{2} \cdot \frac{1}{2} e^x (\cos x + \sin x) \right).$$

Q11 $\int_{-\pi/2}^0 \sin^2(x) \cos^3(x) dx = \int_{-\pi/2}^0 \sin^2(x) (1 - \sin^2(x)) \cos x dx$ sub $u = \sin x$
 $\frac{du}{dx} = \cos x.$

$$= \int_{-1}^0 u^2 (1 - u^2) \cos x \cdot \frac{1}{\cos x} du = \left[\frac{u^3}{3} - \frac{u^5}{5} \right]_{-1}^0 = -\left(-\frac{1}{3} + \frac{1}{5} \right) = +\frac{2}{15}.$$

Q12 $\int \sin(3x) \cos(7x) dx = \frac{1}{2} \int \sin(10x) - \sin(-4x) dx$

we $\left. \begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ \sin(A-B) &= \sin A \cos B - \cos A \sin B \end{aligned} \right\} \sin A \cos B = \frac{1}{2} (\sin(A+B) - \sin(A-B))$

$= \frac{1}{20} \cos(10x) + \frac{1}{8} \sin(4x) + c$

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