(1) Show that the following spaces have the same chomology groups in each dimension, but are not homotopy equivalent.
  (a) $\mathbb{CP}^2$ and $S^2 \vee S^4$
  (b) $\mathbb{RP}^3$ and $\mathbb{RP}^2 \vee S^3$.
(2) Construct explicit co-cycles and cup products for the following CW complexes, with given coefficients, obtained by identifying a polygon using the following words.
  (a) a^5 with $\mathbb{Z}/5\mathbb{Z}$ coefficients.
  (b) a^3b^3 with $\mathbb{Z}/3\mathbb{Z}$ coefficients.
(3) Let $T^2 = S^1 \times S^1$ and K be the the Klein bottle.
  (a) Prove that $f^*\colon H^2(T^2;\mathbb{Z}) \to H^2(S^2;\mathbb{Z})$ is trivial for any map $f\colon S^2 \to T^2$.
  (b) Prove that $f_*\colon H_2(S^2;\mathbb{Z}) \to H_2(T^2;\mathbb{Z})$ is trival.
  (c) Can you say the same about g^* for any map $g\colon T^2 \to S^2$?
  (d) Prove that $f^*\colon H^2(K;\mathbb{Z}/2\mathbb{Z}) \to H^2(S^2;\mathbb{Z}/2\mathbb{Z})$ is trivial for any map $f\colon S^2 \to K$.
  (e) Prove that $f^* : H^2(T^2;\mathbb{Z}/2\mathbb{Z}) \to H^2(K;\mathbb{Z}/2\mathbb{Z})$ is trivial for any map $f\colon K \to T^2$.
  (f) Prove that $f^*\colon H^2(K;\mathbb{Z}/2\mathbb{Z}) \to H^2(T^2;\mathbb{Z}/2\mathbb{Z})$ is trivial for any map $f\colon T^2 \to K$.
(4) Let M_g denote the closed orientable surface of genus $g \geq 1$.
  (a) Prove that for each nonzero $\alpha \in H^1(M_g;\mathbb{Z})$, there exists $\beta \in H^1(M_g;\mathbb{Z})$ with $\alpha \cup \beta \neq 0$.
  (b) Prove that for any map $f\colon M_m \to M_n$ with $n > m$, the map $f^*\colon H^2(M_n) \to H^2(M_m)$ is trivial.
  (c) Prove that M_g is not homotopy equivalent to a wedge sum $X \vee Y$ of CW complexes, each with nontrivial reduced homology.
(5) Prove that a degree 1 map between manifolds induces a surjection on the fundamental groups.

Also: Hatcher §3.2 Q 3,8.