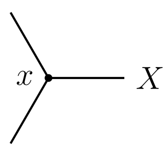
A sequence of abelian groups and homomorphisms is *exact* if the image of one map is the kernel of the next. (i.e. a chain with trivial homology).

(1) For each of the following exact sequences of abelian groups and homomorpohisms, say as much as you can about the unknown group G, and/or the unknown homomorphism α.
  (a) $0 \to \mathbb{Z}/2 \to G \to \mathbb{Z} \to 0$
  (b) $0 \to \mathbb{Z} \to G \to \mathbb{Z}/2 \to 0$
  (c) $0 \to \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z} \oplus \mathbb{Z}/2 \to 0$
  (d) $0 \to G \xrightarrow{\alpha} \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z}/2 \to 0$
  (e) $0 \to \mathbb{Z}/3 \to G \to \mathbb{Z}/2 \to \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z} \to 0$

(2) Compute the local homology groups $H_*(X, X \setminus x)$, where x is the central vertex of the graph consisting of three edges meeting at a single vertex.



(3) Describe explicit cell structures on the following spaces.
  (a) The union of the unit sphere in $\mathbb{R}^3$ with the parts of the x- and y-axes contained in the unit ball.
  (b) The union of two round spheres in $\mathbb{R}^3$ which intersect in a single circle.
  (c) The union of the closed unit ball in $\mathbb{R}^3$ with the closed ball of radius 2 in the xy-plane.

Also: Hatcher §2.1 Q14, 20