Math 229 Calculus Computer Lab Spring 15 Midterm 2a

Name: _____Solutions_____

- I will count your best 5 of the following 6 questions.
- You may only use julia during this exam. No calculators or cell phones or notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
	50	

Midterm 1	
Overall	

(1) Convert the following `julia` expressions to standard mathematical expressions. Use parentheses to clearly indicate the order of operations:

(a) `a-b/(c-a)`

$$a - \frac{b}{c-a}$$

(b) `x+y/2z+3`

$$x + \frac{y}{2z} + 3$$

(c) `tan(x)^2/2x/e^x*x`

$$\frac{\tan^2(x)\, x}{2x\, e^x}$$

(2) Is e^x^3 equal to $e^{(x^3)}$ or $(e^x)^3$? Write julia commands which show your answer is correct.

$$e^{(x^3)}$$

$$e^{\wedge}1^{\wedge}3 = 2.718\ldots$$

$$e^{\wedge}(1^{\wedge}3) = 2.718\ldots$$

$$(e^{\wedge}1)^{\wedge}3 = 20.085\ldots$$

(3) Write `julia` commands to find the zeros of $f(x) = x^2 - 2x + 1$ using

(a) `roots`

$f(x) = x^2 - 2x + 1$

using Roots

roots(f) : 1.0
1.0

(b) `fzeros`
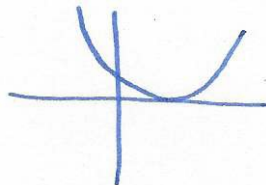
fzeros(f) : 1.0

(c) The bisection method `fzero`

fzero(f, -10, 10)

The last one doesn't work - explain why.
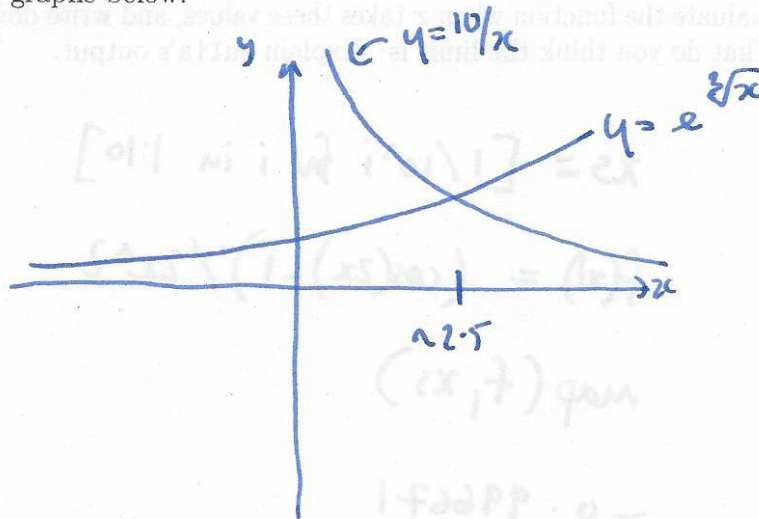
$f(x) = (x-1)^2$

doesn't cross x-axis
bisection method needs an interval
for which the endpoints give values of
f with different signs.

(4) Consider the equation $e^{\sqrt[3]{x}} = 10/x$.

(a) Show there is a solution by plotting the graphs of these functions. Sketch the graphs below.

(b) Write `julia` commands to find a numerical approximation to the solution, and find the solution.

$$f(x) = e\char`\^(x\char`\^(1.0/3)) - 10/x$$

$$\text{fzero}(f, [2,3])$$

$$x = 2.550\cdots$$

(5) You wish to estimate

$$\lim_{x \to 0} \frac{\cos(2x) - 1}{2x^2}.$$

Write julia commands to generate a list of numbers $\{10^{-1}, 10^{-2}, \ldots, 10^{-10}\}$. Evaluate the function when x takes these values, and write down your results. What do you think the limit is? Explain julia's output.

```
xs = [1/10^i for i in 1:10]

f(x) = (cos(2x)-1)/2x^2

map(f, xs)
```

-0.996671

-0.999967

-1.0

-1.0

-1.0

-0.999977

-0.999201

-1.11022

0.0

0.0

limit is -1

$\cos(2x) \to 1$ as $x \to 0$, so numerator is difference of two numbers close to 1, and float numbers have a fixed accuracy, so when $\cos(2x)$ is too close to 1, $\cos(2x) - 1$ becomes zero.

(6) Find $\lim_{x \to 0} e^{-1/x^2}$ by any method. Write down the `julia` commands you use.

using SymPy

$f(x) = e^{\wedge}(-1/x^{\wedge}2)$

limit (f, 0)

0