**Math 232 Calculus 2 Spring 15 Midterm 1a**

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a $3 \times 5$ index card of notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 1 | |
| Overall | |

(1) (10 points) Find $\int \frac{e^{2x}}{1-e^{2x}}\,dx$.

$u = 1-e^{2x}$

$\frac{du}{dx} = -2e^{2x}$

$\int \frac{e^{2x}}{u} \frac{dx}{du}\,du$

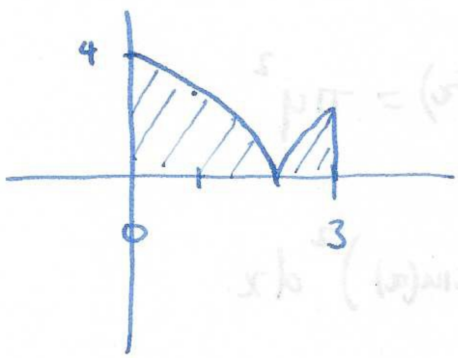
$\int \frac{e^{2x}}{u} \cdot \frac{1}{-2e^{2x}}\,du$

$-\frac{1}{2}\int \frac{1}{u}\,du$

$-\frac{1}{2}\ln|u| + c$

$-\frac{1}{2}\ln|1-e^{2x}| + c$

(2) (10 points) Find $\int_0^3 \left|x^2 - 4\right| \, dx$. Draw a picture of the region.




$x^2 - 4 = 0$

$(x-2)(x+2) = 0 \qquad x = \pm 2$

$\int_0^2 -x^2 + 4 \, dx + \int_2^3 x^2 - 4 \, dx$

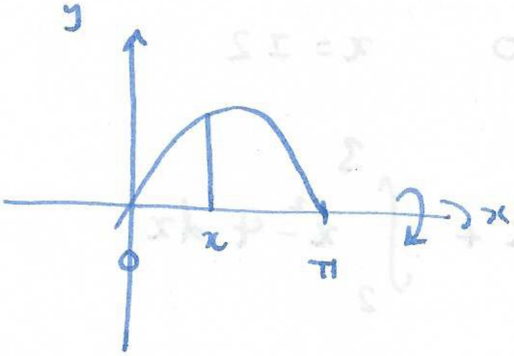
$= \left[ -\frac{1}{3}x^3 + 4x \right]_0^2 + \left[ \frac{1}{3}x^3 - 4x \right]_2^3$

$-\frac{8}{3} + 8 \qquad + 9 - 12 - \left( \frac{8}{3} - 8 \right)$

$\frac{16}{3} + 5 - \frac{8}{3}$

$7\frac{2}{3} = \frac{23}{3}$.

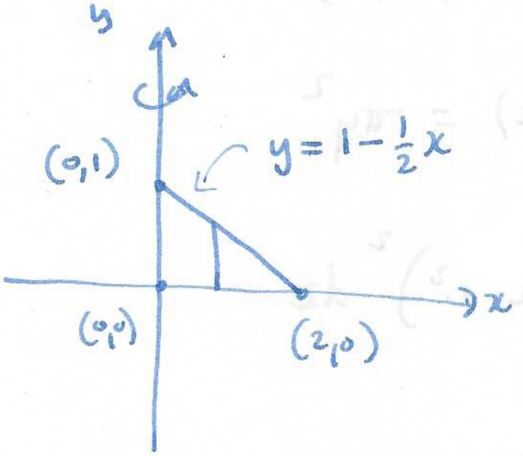
(3) (10 points) Draw a picture of the region bounded by the curve $y = 3\sin(x)$, for $0 \leq x \leq \pi$ and $y \geq 0$. Write down an integral to give you the volume of revolution of this region about the $x$-axis. DO NOT EVALUATE THIS INTEGRAL.




disks: $A(x) = \pi y^2$

$$\int_0^{\pi} \pi \left(3\sin(x)\right)^2 dx$$

(4) (10 points) Use shells to write down an integral for the volume of the cone formed by rotating the triangle with vertices $(0,0)$, $(0,1)$ and $(2,0)$ about the $y$-axis. DO NOT EVALUATE THIS INTEGRAL.

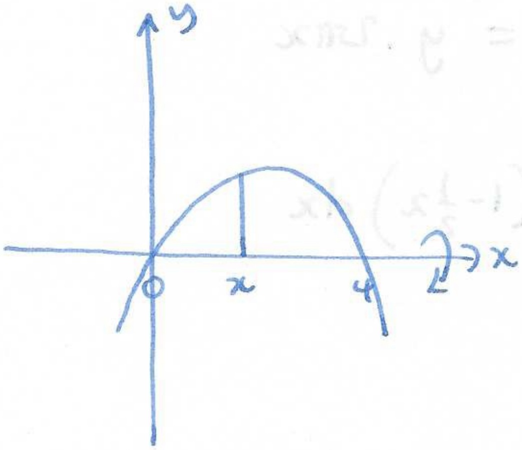



shells $A(x) = y \cdot 2\pi x$

$$\int_0^2 2\pi x\left(1 - \tfrac{1}{2}x\right) dx$$

(5) (10 points) Consider the subset of the plane bounded by $y = 4x - x^2$ in the first quadrant (i.e. $x \geq 0$ and $y \geq 0$). Find the volume of revolution of the 3-dimensional shape formed by rotating this region around the $x$-axis.

$y = 4x - x^2 = x(4-x)$




discs: $A(x) = \pi y^2$

$$\int_0^4 \pi (4x - x^2)^2 \, dx$$

$$= \pi \int_0^4 16x^2 - 8x^3 + x^4 \, dx$$

$$\pi \left[ \frac{16}{3}x^3 - 2x^4 + \frac{1}{5}x^5 \right]_0^4$$

$$\pi \left( \frac{1024}{3} - 512 + \frac{1024}{5} \right)$$

$$\pi \left(34\tfrac{1}{6}\right) \simeq 107.33$$

(6) (10 points) Find $\int \underbrace{\sqrt{x}}_{v'}\, \underbrace{\ln(4x)}_{u}\, dx$.

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = \ln(4x) \qquad u' = \frac{1}{x}$

$v' = x^{1/2} \qquad v = \frac{2x^{3/2}}{3}$

$$= \frac{2x^{3/2}}{3}\ln(4x) - \int \frac{1}{x}\cdot\frac{2}{3}x^{3/2}\,dx$$

$$\frac{2}{3}x^{3/2}\ln(4x) - \int \frac{2}{3}x^{1/2}\,dx$$

$$\frac{2}{3}x^{3/2}\ln(4x) - \frac{4}{9}x^{3/2} + c$$

$$= \frac{2}{3}x^{3/2}\ln(4x) - \frac{4}{9}x^{3/2} + c$$

(7) (10 points) Find $\int \underbrace{e^{-3x}}_{u} \underbrace{\sin(x)}_{v} \, dx$.

$$\int uv' \, dx = \int uv - \int u'v \, dx$$

$u = e^{-3x}$ $\quad u' = -3e^{-3x}$

$v' = \sin x$ $\quad v = -\cos x$

$$\int e^{-3x} \sin(x) \, dx = -e^{-3x} \cos x - \int \underbrace{3e^{-3x}}_{u} \underbrace{\cos x}_{v} \, dx$$

$u = 3e^{-3x}$ $\quad u' = -9e^{-3x}$

$v' = \cos x$ $\quad v = \sin x$

$$\int e^{-3x} \sin(x) \, dx = -e^{-3x} \cos x - 3e^{-3x} \sin x - \int 9e^{-3x} \sin x \, dx$$

$$10 \int e^{-3x} \sin(x) \, dx = -e^{-3x} \cos x - 3e^{-3x} \sin x + c$$

$$\int e^{-3x} \sin(x) \, dx = -\frac{1}{10} e^{-3x} \cos x - \frac{3}{10} e^{-3x} \sin x + c$$

(8) Find $\int_0^{\pi/2} \cos^5 x \, dx$. $= \int_0^{\pi/2} \cos^4 x \cos x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$$= \int_0^{\pi/2} (1 - \sin^2 x)^2 \cos x \, dx$$

$$= \int_0^{1} (1-u^2)^2 \cos x \frac{dx}{du} \, du$$

$$= \int_0^{1} (1 - 2u^2 + u^4) \cos x \frac{1}{\cos x} \, du$$

$$= \left[ u - \frac{2}{3}u^3 + \frac{1}{5}u^5 \right]_0^1$$

$$= 1 - \frac{2}{3} + \frac{1}{5} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

(9) Find $\int \sin(2x)\sin(5x)\,dx$.

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$

$$= \int \frac{1}{2}\cos(2x-5x) - \frac{1}{2}\cos(2x+5x)\,dx$$

$$= \frac{1}{2}\int \cos 3x - \cos 7x \, dx$$

$$= \frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + c$$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

(10) Find $\int \sqrt{4 - x^2}\, dx$.

$x = 2\sin u$

$\frac{dx}{du} = 2\cos u$

$$\int \sqrt{4 - 4\sin^2 u} \cdot \frac{dx}{du}\, du$$

$$2\int \sqrt{1 - \sin^2 u}\; 2\cos u\, du$$

$$2\int \sqrt{\cos^2 u}\; 2\cos u\, du$$

$$4\int \cos^2 u\, du$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$

$$4\int \frac{1}{2}\cos 2u + \frac{1}{2}\, du$$

$$2\left[\frac{1}{2}\sin 2u + \frac{4}{4}u\right]$$

$$\sin 2u + 2u + C$$

$$2\sin u\cos u + 2u + C$$

$\sin u = \frac{x}{2}$

$\cos u = \sqrt{1 - x^2/4}$.

$$\frac{2x}{2}\sqrt{1 - x^2/4} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$$