

Math 330 ODEs Fall 15 Linear Algebra Fun sheet

- (1) Describe the geometric action of the following matrices as maps from $\mathbb{R}^2 \rightarrow \mathbb{R}^2$.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(e) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(f) $\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(h) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(j) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(k) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

- (2) Decide which of the following geometric transformations $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ correspond to linear maps, and find the corresponding matrices.

(a) Translation by $\langle 1, 0 \rangle$.

(b) Expansion by a factor of 2 in all directions about the origin.

(c) Reflection in the x -axis.

(d) Reflection in $y = x$.

(e) Projection on to the y -axis.

(f) Rotation by $\pi/2$ radians around the origin.

(g) Rotation by θ radians around the origin.

- (3) Describe the geometric action of the following matrices as maps from $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ or $\mathbb{R}^3 \rightarrow \mathbb{R}^3$.

(a)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d)
$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (4) Decide which of the following geometric transformations $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ correspond to linear maps, and find the corresponding matrices.

(a) Translation by $\langle 1, 0, 0 \rangle$.

(b) Expansion by a factor of 2 in all directions about the origin.

(c) Reflection in the xy -plane.

(d) Projection on to the yz -plane.

(e) Rotation by $\pi/2$ radians around the x -axis.

(f) Rotation by θ radians around the y -axis.