Math 233 Calculus 3 Fall 15 Sample midterm 1

1. Let $\vec{u} = \langle -2, 3, 4 \rangle$ and $\vec{v} = \langle 1, -2, 3 \rangle$.

   (a) Find $||\text{proj}_{\vec{v}}\vec{u}||$.

   (b) Express $\vec{u}$ as the sum of $\vec{m} = \vec{u}_{||}$ parallel to $\vec{v}$, and $\vec{n} = \vec{u}_{\perp}$ orthogonal to $\vec{v}$.

2. Consider three points $A(-1, -2, 2)$, $B(1, 2, -2)$, $C(3, 1, 2)$.

   (a) Find the area of the triangle formed by A, B, C.

   (b) Find the equation of the plane that contains A, B, C.

3. Find the equation of the plane that passes through the point $(5, -7, 4)$ and is perpendicular to the line $\langle 6, -3, -1 \rangle + t\langle 2, -1, -2 \rangle$.

4. For each equation below, sketch the surface in $\mathbb{R}^3$ that it describes.

   (a) $x^2 + 4y^2 + 4z^2 = 16$

   (b) $z = 9x^2 - 4y^2$

   (c) $4x^2 + 9z^2 = 2y^2 - 72$

   (d) $9x^2 - 4y^2 = 72$

5. A particle starts at location $\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ with initial velocity $2\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$. Its acceleration is $\boldsymbol{a}(t) = 3\boldsymbol{i} + 12t^2\boldsymbol{j} - 6t\boldsymbol{k}$. Find the location of the particle at $t = 2$.

6. A string in the shape of a helix has a height of 12cm and makes four full rotations over a circle of radius 3cm.

   (a) Find a parametrization $\boldsymbol{r}(t)$ for the string.

   (b) Compute the length of the string.

7. Show that the following limit does not exist.
$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{x^2 + y^2}$$

8. Find all the second order partial derivatives of
$$f(x, y, z) = ye^{xy+z} + (x + z)\tan(y + z)$$
   You may assume that mixed partials are equal.

9. Find the equation of the tangent plane to the surface $z = 2x^2 - 4y^2$ at the point $(1, 1, -2)$.

10. Find the linear approximation to the function $f(x, y, z) = e^{4yz} + \ln(x + z)$ at the point $(4, -2, 1)$.

11. Find the normal vector to the surface $z^2 = 3x^2 - 2y^2$ at the point $(3, -1, 5)$.