

A sequence of abelian groups and homomorphisms is *exact* if the image of one map is the kernel of the next. (i.e. a chain with trivial homology).

For each of the following exact sequences of abelian groups and homomorphisms, say as much as you can about the unknown group G , and/or the unknown homomorphism α .

$$(1) 0 \rightarrow \mathbb{Z}/2 \rightarrow G \rightarrow \mathbb{Z} \rightarrow 0$$

$$(2) 0 \rightarrow \mathbb{Z} \rightarrow G \rightarrow \mathbb{Z}/2 \rightarrow 0$$

$$(3) 0 \rightarrow \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z} \oplus \mathbb{Z} \rightarrow \mathbb{Z} \oplus \mathbb{Z}/2 \rightarrow 0$$

$$(4) 0 \rightarrow G \xrightarrow{\alpha} \mathbb{Z} \oplus \mathbb{Z} \rightarrow \mathbb{Z}/2 \rightarrow 0$$

$$(5) 0 \rightarrow \mathbb{Z}/3 \rightarrow G \rightarrow \mathbb{Z}/2 \rightarrow \mathbb{Z} \xrightarrow{\alpha} \mathbb{Z} \rightarrow 0$$