MATH 341 - FINAL EXAM

College of Staten Island Spring 2013 Pribitkin

Instructions: You are permitted three $8\frac{1}{2}'' \text{ X } 11''$ sheets of paper with your own handwritten/typed notes on both sides. Calculators are allowed, but all answers must be explained (just as we did in class). Please write neatly. The time limit is 115 minutes. Good luck!

1. (3 points each) Answer each of the following as true or false.

   a. Between any two distinct irrational numbers there is a rational number.

   b. Every ordered field satisfies the Least Upper Bound Property.

   c. If the sequence $\{s_n\}_{n=1}^{\infty}$ is nonincreasing and bounded above, then it converges.

   d. If f is differentiable at x_0, then f' is continuous at x_0.

   e. If f is differentiable on $[a, b]$, then f is antidifferentiable on $[a, b]$.

   f. If $f + g$ is integrable on $[a, b]$, then both f and g are bounded on $[a, b]$.

   g. Let $\Delta x = (b - a)/n$, where $n \in \mathbb{N}$. If $\lim\limits_{\Delta x \to 0} \sum_{k=1}^{n} f(a + k\Delta x)\, \Delta x$ exists, then f is integrable on $[a, b]$.

   h. If f has a jump discontinuity somewhere on $[a, b]$, then f is not antidifferentiable on $[a, b]$.

   i. If both f and g are differentiable on $[a, b]$ and $g'(x) \neq 0$ on (a, b), then there exists at least one number $X \in (a, b)$ such that $\dfrac{f(b) - f(a)}{g(b) - g(a)} = \dfrac{f'(X)}{g'(X)}$.

   j. If both f and g are continuous on $[a, b]$, then there exists at least one number $X \in [a, b]$ such that $\int_a^b f(x)g(x)\, dx = f(X) \int_a^b g(x)\, dx$.

   k. It is possible to find Taylor's Formula with Remainder for $f(x) = e^{|x|}$, $x \in [-1, 1]$, at $c = 0$.

   l. ∞^0 is an indeterminate form.

2. (6 points each) Find each of the following. *Explain* your answers.

   a. $f'(1)$ for $f(x) = (x - 1)\,|\log x|$

   b. $\displaystyle\bigcap_{n=1}^{\infty} [\cos(1/n), \sec(1/n)]$

   c. An upper bound on $|g(x)|$, where $g(x) = \dfrac{-x^4 - 16}{5 - \sin(\pi/x)}$, $0 < x \leq 2$.

   d. $\lim\limits_{x \to +\infty} x^{1/x}$

   e. $\lim\limits_{x \to +\infty} \dfrac{\int_0^x e^{\cos(\sin t)}\, dt}{e^x + x^3}$

3. (10 points) Let f be continuous on $[0, 2]$. Show that there exists an $X \in [0, 2]$ such that $\displaystyle\int_0^2 f(x)x^3\, dx = 4f(X)$.

4. (12 points) Let g be continuous on $[-1, 1]$ and differentiable on $(-1, 1)$. Also assume that $g(-1) = 0$, $g(0) = 1$, and $g(1) = 0$. Prove that there exists a number $c \in (-1, 1)$ such that $g'(c) = 1/2$.

5. (12 points) Which number is larger, $e^{2.7182818285}$ or 2.7182818285^e? Or are they equal? Prove your assertion.

EXTRA CREDIT: Find $\lim\limits_{x \to +\infty} \left[(x^n + Ax^{n-1})^{1/n} - x\right]$, where $n \in \mathbb{N}$ and $A \in \mathbb{R}$. *Justify* your answer.