MTH 341
SUMMER 2009

FINAL EXAM

Solve **9 OUT OF 10** problems. Place all solutions in the exam book. **Show all work**!

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1. Use the ϵ, δ definition to prove that $\lim_{x\to 2}(x^2+1) = 5$.

2. Find a function $F(x)$ such that $F'(x) = e^{-x^2}$.

3. State and prove the Mean-Value Theorem.

4. Use the Mean-Value Theorem to prove that $\sin x < x$, for $x > 0$

5. Show directly that $\lim_{n\to\infty}\left(\frac{n+1}{2n^2+1}\right) = 0$.

6. Consider the set $A = \left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$. Show in detail that $\sup(A) = 1$.

7. Prove that the recursive sequence defined by $x_1 = 1$, and $x_{n+1} = \frac{2x_n + 3}{4}$ for $n \geq 2$, converges and find its limit.

8. Use the limit definition of the integral to prove that $\int_0^1 2x dx = 1$.

9. State both forms of the Archimedean Property.

10. Use the ϵ, δ definition to prove that $f(x) = mx + b$, where $m \neq 0$ and b are real constants, is continuous everywhere.