Advice to the beginner

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Mathematics is the backbone of modern science and a remarkably efficient source of new concepts and tools to understand the "reality" in which we participate. The new concepts themselves are the result of a long process of "distillation" in the alembic of the human thought.

I was asked to write some advice for young mathematicians. The first observation is that each mathematician is a special case, and in general mathematicians tend to behave like "fermions" *i.e.* avoid working in areas which are too trendy whereas physicists behave a lot more like "bosons" which coalesce in large packs and are often "overselling" their doings, an attitude which mathematicians despise.

It might be tempting at first to view mathematics as the union of separate parts such as Geometry, Algebra, Analysis, Number theory etc... where the first is dominated by the understanding of the concept of "space", the second by the art of manipulating "symbols", the next by the access to "infinity" and the "continuum" etc...

This however does not do justice to one of the most essential features of the mathematical world, namely that it is virtually impossible to isolate any of the above parts from the others without depriving them from their essence. In that way the corpus of mathematics does resemble a biological entity which can only survive as a whole and would perish if separated into disjoint pieces.

The scientific life of mathematicians can be pictured as a trip inside the geography of the "mathematical reality" which they unveil gradually in their own private mental frame.

It often begins by an act of rebellion with respect to the existing dogmatic description of that reality that one will find in existing books. The young "to be mathematician" realize in their own mind that their perception of the mathematical world captures some features which do not quite fit with the existing dogma. This first act is often due in most cases to ignorance but it allows one to free oneself from the reverence to authority by relying on one's intuition provided it is backed up by actual proofs. Once mathematicians get to really know, in an original and "personal" manner, a small part of the mathematical world, as esoteric as it can look at first[1], their trip can really start. It is of course vital all along not to break the "fil d'arianne" which allows to constantly keep a fresh eye on whatever one will encounter along the way, and also to go back to the source if one feels lost at times...

It is also vital to always keep moving. The risk otherwise is to confine oneself in a relatively small area of extreme technical specialization, thus shrinking one's perception of the mathematical world and of its bewildering diversity.

The really *fundamental point* in that respect is that while so many mathematicians have been spending their entire scientific life exploring that world they all agree on its contours and on its connexity: whatever the origin of one's itinerary, one day or another if one walks long enough, one is bound to reach a well known town *i.e.* for instance to meet elliptic functions, modular forms, zeta functions. "All roads lead to Rome" and the mathematical world is "connected". Of course this is not to say that all parts of mathematics look alike and it is worth quoting what Grothendieck says (in "Récoltes et semailles") in comparing the landscape of analysis in which he first worked with that of algebraic geometry in which he spent the rest of his mathematical life:

> "Je me rappelle encore de cette impression saisissante (toute subjective certes), comme si je quittais des steppes arides et revêches, pour me retrouver soudain dans une sorte de "pays promis" aux richesses luxuriantes, se multipliant à l'infini partout où il plait à la main de se poser, pour cueillir ou pour fouiller...."

Most mathematicians adopt a pragmatic attitude and see themselves as the explorers of this "mathematical world" whose existence they don't have any wish to question, and whose structure they uncover by a mixture of intuition, not so foreign from "poetical desire"[2], and of a great deal of

[1] my starting point was localization of roots of polynomials, but I was fortunately invited at a very early age in a conference in Seattle where I found the roots of all my future work on factors.

[2] as emphasised by the French poet Paul Valery.

rationality requiring intense periods of concentration.

Each generation builds a "mental picture" of their own understanding of this world and constructs more and more penetrating mental tools to explore previously hidden aspects of that reality.

Where things get really interesting is when unexpected bridges emerge between parts of the mathematical world that were previously believed to be very far remote from each other in the natural mental picture that a generation had elaborated. At that point one gets the feeling that a sudden wind has blown out the fog that was hiding parts of a beautiful landscape. In my own work this type of "great surprise" came mostly from the interaction with physics. The depth of mathematical concepts that come directly from physics has been qualified in the following terms by Hadamard :

> "Not this short lived novelty which can too often influence the mathematician left to his own devices, but this infinitely fecund novelty which springs from the nature of things"

I'll end by some more "practical" advise[3],

- **Walks**

  One very sane exercise, when fighting with a very complicated problem (often involving computations), is to go for a long walk (no paper or pencil) and do the computation in one's head (irrespective of the first feeling "it is too complicated to be done like that"). Even if one does not succeed it trains the "live memory" and sharpens the skills.

- **Lying down**

  Mathematicians usually have a hard time explaining to their partner that the times when they work with most intensity is when they are lying down in the dark on a sofa. Unfortunately with email and the invasion of computer screens in all math. institutes this manner to isolate oneself and concentrate tends to become more rare, it is all the more valuable.

- **Being brave**

  There are several phases in the process leading to "find" new math. and while the "checking" phase is scary and involves just rationality and concentration, the "creative" first phase is of a totally different nature. In some sense it requires a kind of protection of one's ignorance since there are always billions of rational reasons not to look at a problem which has been unsuccessfully looked at by generations of mathematicians.

- **Set backs**

  It does often happen in the life of a mathematician and at any stage (often very early on) of their scientific life, that one for instance gets a "preprint" of a competitor and feels disrupted. The only recipe I have there is to "try" (it is not always easy) to convert this feeling of frustration into positive energy for working harder.

- **Grudging Approbation**

  A colleague of mine once said "We[4] work for the grudging approbation of a few friends". It is true that since the research work is of a rather solitary nature we need badly that approbation in a way or another, but quite frankly don't expect much.... In fact there is no way to fool around with the only real judge which is oneself, and caring too much about the opinion of others is a waste of time: so far no theorem has been proved as a result of a vote. As Feynman put it "why do you care what other people think"!

[3]recalling that each mathematician is a "special case" don't take the advice too literally.

[4]mathematicians