Math 130 Precalculus Fall 14 Midterm 3b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Solve

(a) $2e^{2x} + e^x = 1$

$2(e^x)^2 + e^x - 1$

$(2e^x - 1)(e^x + 1)$

$e^x = \frac{1}{2}$ $\quad e^x = -1$ no solutions

$x = \ln(1/2)$

$x = -\ln(2)$

(b) $\ln(2x) = \ln(3x + 2) + 1$

$\ln\left(\frac{2x}{3x+2}\right) = 1$

$\frac{2x}{3x+2} = e$

$2x = 3ex + 2e$

$x(2 - 3e) = 2e$

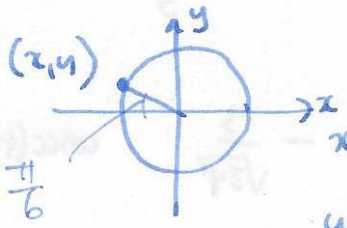
$x = \frac{2e}{2 - 3e}$

(2) Find the point on the unit circle

(a) corresponding to the terminal point for $t = -31\pi/6$.

$\frac{31}{6} = 5 + \frac{1}{6}$

$= 2 + 1 + \frac{1}{6}$



$x = \cos t = -\frac{\sqrt{3}}{2}$

$y = \sin t = \frac{1}{2}$

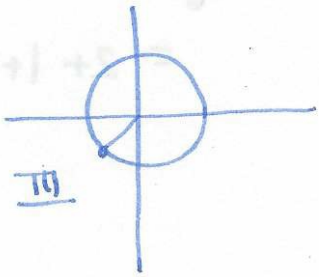
(b) whose y-coordinate is $-3/5$ and whose x-coordinate is positive.

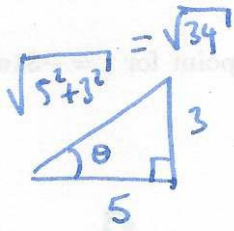
$x^2 + y^2 = 1$

$x^2 + \frac{9}{25} = 1 \qquad x^2 = 1 - \frac{9}{25} = \frac{16}{25}$

$x = \pm \frac{4}{5} \qquad x = \frac{4}{5}$.

(3) If $\tan(t) = 3/5$ and t is in quadrant III find the values of the other trig functions at t.





$\cot(t) = \frac{5}{3}$

$\sin t = -\frac{3}{\sqrt{34}}$ $\quad \operatorname{csc}(t) = -\frac{\sqrt{34}}{3}$

$\cos t = -\frac{5}{\sqrt{34}}$ $\quad \sec(t) = -\frac{\sqrt{34}}{5}$

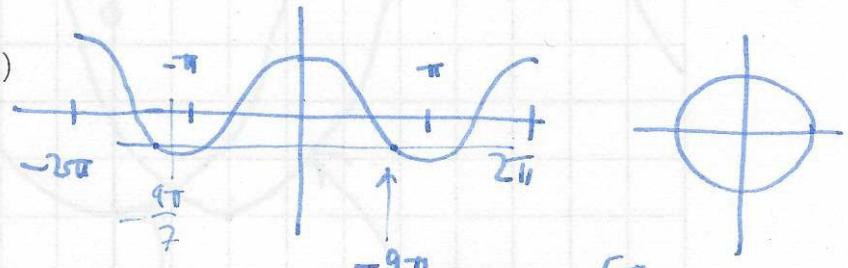
(4) Find the exact value of

(a) $\tan(21\pi/4)$

$\frac{21}{4} = 5 + \frac{1}{4}$

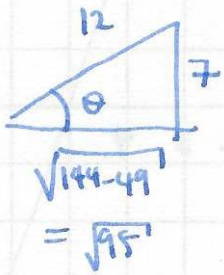
$= \tan\left(\frac{\pi}{4}\right) = 1$

(b) $\cos^{-1}(\cos(-9\pi/7))$



$-\frac{9\pi}{7} + 2\pi = \frac{5\pi}{7}$

(c) $\tan(\sin^{-1}(7/12))$

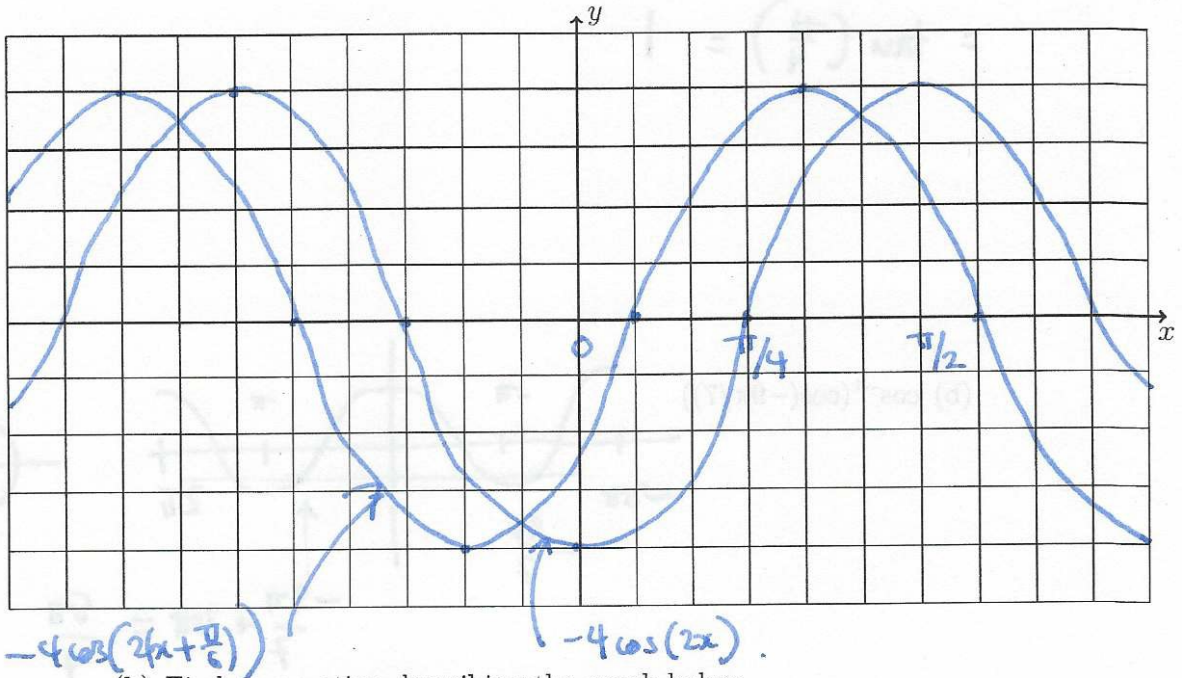


$\tan\theta = \frac{7}{\sqrt{95}}$

(5) (a) Find the amplitude, frequency and phase shift for $y = -4\cos(2x + \pi/3)$, and draw a careful graph of the function below.

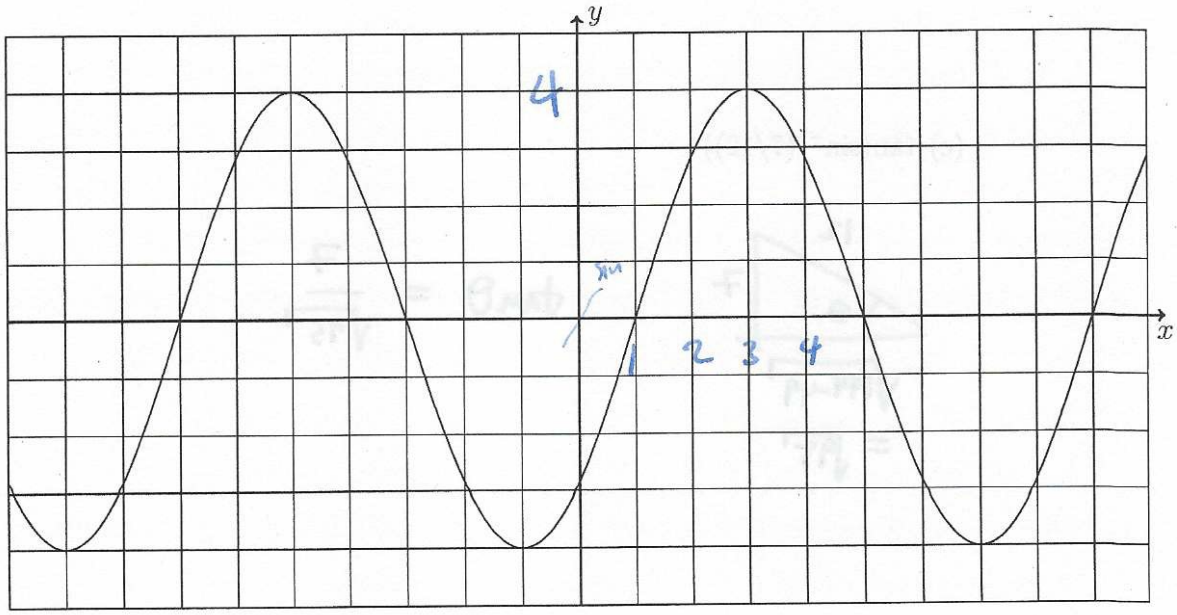
Period $= \frac{2\pi}{h} = \pi$

$-4\cos\left(2\left(x+\frac{\pi}{6}\right)\right)$



$-4\cos\left(2x+\frac{\pi}{6}\right)$ $\quad -4\cos(2x)$.

(b) Find an equation describing the graph below.



period $= 8 = \frac{2\pi}{h}$

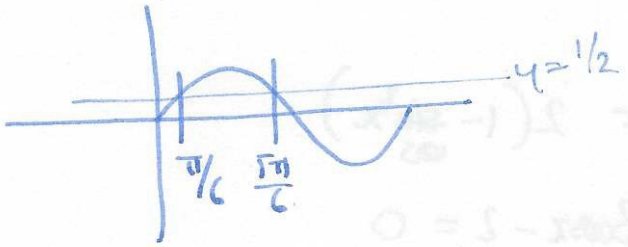
$h = \frac{\pi}{4}$

$4\sin\left(\frac{\pi}{4}(x-1)\right)$

(6) Find all solutions to

$$2\sin(x) = 1.$$

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6} + 2\pi n$

$\frac{5\pi}{6} + 2\pi n$

(7) Find all solutions to

$$3\cos(x) = 2\sin^2(x).$$

~~(Hint: you may use the answer to the previous question.)~~

$3\cos x = 2(1 - \sin^2 x)$

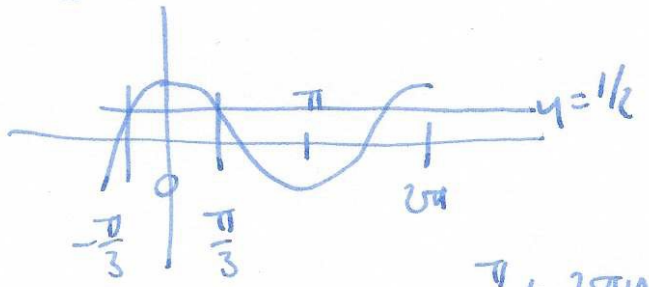
$2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$\cos x = 2$

no solution

$\cos x = \frac{1}{2}$



$\frac{\pi}{3} + 2\pi n$

$-\frac{\pi}{3} + 2\pi n.$

(8) Find all solutions to

$$\sin(2x) + \sin(x) = 0.$$

$2\sin x\cos x + \sin x = 0$

$\sin x\,(2\cos x + 1) = 0$

$\sin x = 0$

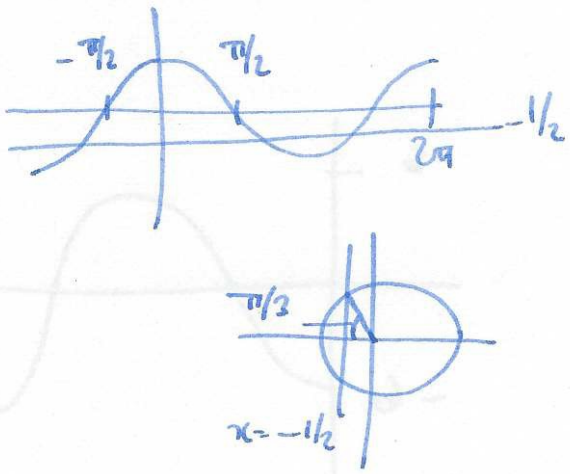
$x = 0 + 2\pi n$

$\pi + 2\pi n$

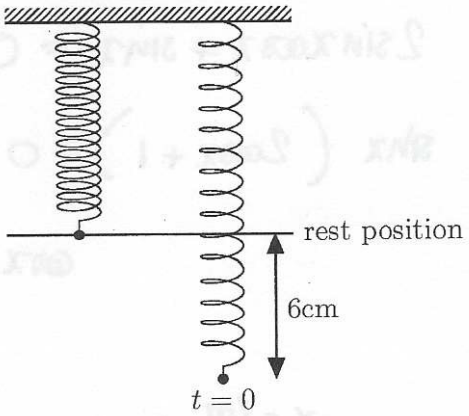
$\cos x = -1/2$

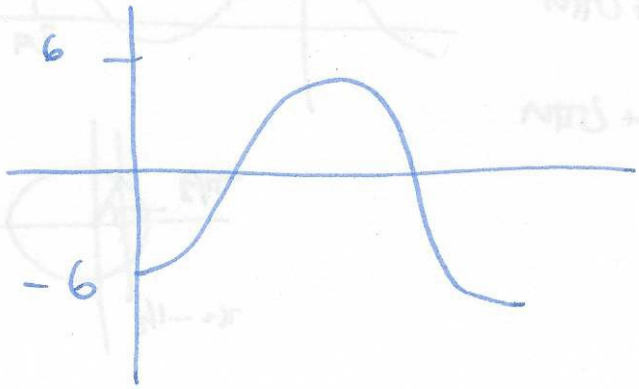
$x = \frac{2\pi}{3} + 2\pi n$

$x = -\frac{2\pi}{3} + 2\pi n$



(9) A spring is pulled down 6cm from its rest position and then released. If it moves according to simple harmonic motion and takes $\frac{1}{2}$ of a second to reach its highest point, find an equation for the height of the spring.





period $= 1 = \frac{2\pi}{k}$

$k = 2\pi$

$-6\cos(2\pi t)$

(10) Prove the identity

$$\frac{\tan x}{\sec x + 1} = \frac{\sec x - 1}{\tan x}.$$

$$\frac{\tan x}{\sec x + 1} \cdot \frac{\sec x - 1}{\sec x - 1} = \frac{\tan x (\sec x - 1)}{\sec^2 x - 1}$$

use: $\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$$= \frac{\tan x (\sec x - 1)}{\tan^2 x}$$

$$= \frac{\sec x - 1}{\tan x}.$$