

# Math 130 Precalculus Fall 14 Midterm 2a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

|    |    |  |
|----|----|--|
| 1  | 10 |  |
| 2  | 10 |  |
| 3  | 10 |  |
| 4  | 10 |  |
| 5  | 10 |  |
| 6  | 10 |  |
| 7  | 10 |  |
| 8  | 10 |  |
| 9  | 10 |  |
| 10 | 10 |  |
|    | 80 |  |

|           |  |
|-----------|--|
| Midterm 2 |  |
| Overall   |  |

- (1) (10 points) Find the ~~minimum~~ <sup>maximum</sup> value of the function  $f(x) = x - 3 - 2x^2$  by completing the square.

$$\begin{aligned}
 -2x^2 + x - 3 &= -2\left(x^2 - \frac{1}{2}x\right) - 3 \\
 &= -2\left(\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right) - 3 = -2\left(x - \frac{1}{4}\right)^2 + \frac{1}{8} - 3 \\
 &= -2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right) - 3
 \end{aligned}$$

max value is  $-3 + \frac{1}{8} = -\frac{23}{8}$

|  |           |
|--|-----------|
|  | Algebra 2 |
|  | (100%)    |

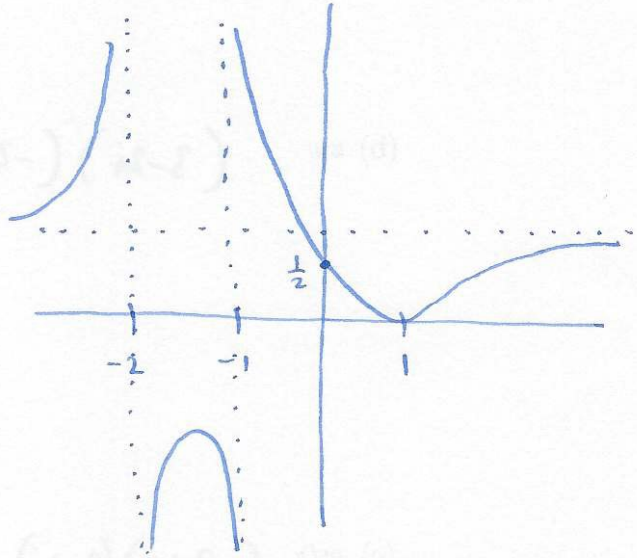
- (2) (10 points) Sketch the graph of the function  $f(x) = \frac{(x-1)^2}{x^2+3x+2}$ . Include the  $x$ - and  $y$ -intercepts, and the vertical and horizontal asymptotes.

$y$  intercept:  $f(0) = \frac{(-1)^2}{2} = \frac{1}{2}$        $x$ -intercepts:  $(x-1)^2 = 0 \Rightarrow x=1$

vertical asymptotes:  $x^2+3x+2 = (x+2)(x+1)$       horizontal asymptote:  $\frac{x^2}{x^2} = 1$   
 $x = -2, x = -1$

sign:

|           |   |   |   |   |
|-----------|---|---|---|---|
| $(x-1)^2$ | + | + | + | + |
| $(x+1)$   | - | - | + | + |
| $(x+2)$   | - | + | + | + |
| $f(x)$    | + | - | + | + |



(3) (10 points) Let  $z = 3 - 2i$  and let  $w = -2 + i$ . Write the following complex numbers in the form  $a + bi$ , where  $a$  and  $b$  are real numbers.

(a)  $w - 2z$

$$\begin{aligned} -2 + i - 2(3 - 2i) &= -2 - 6 + i + 4i \\ &= -8 + 5i \end{aligned}$$

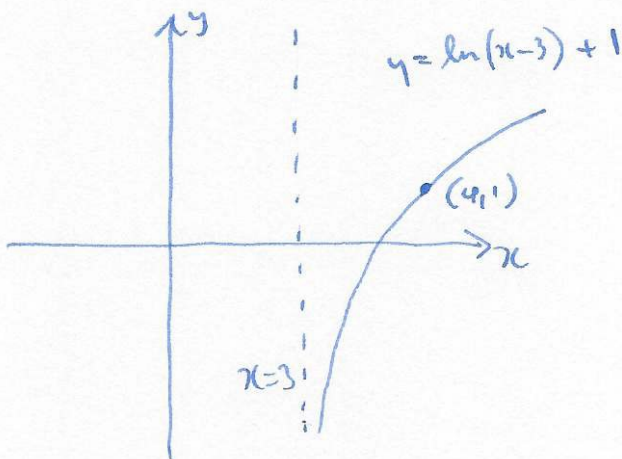
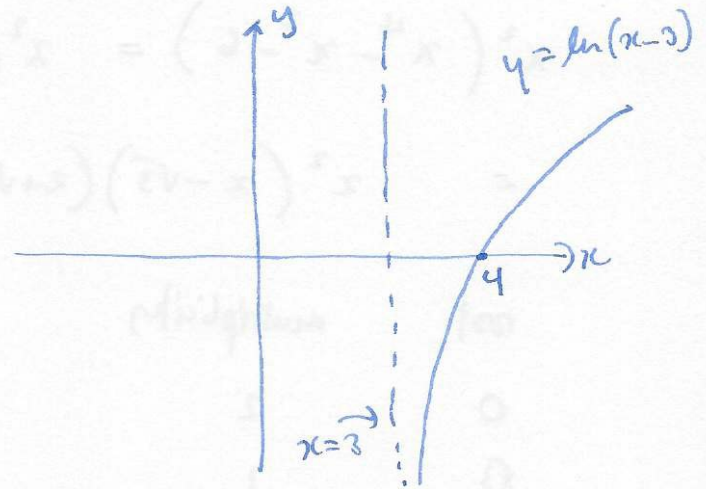
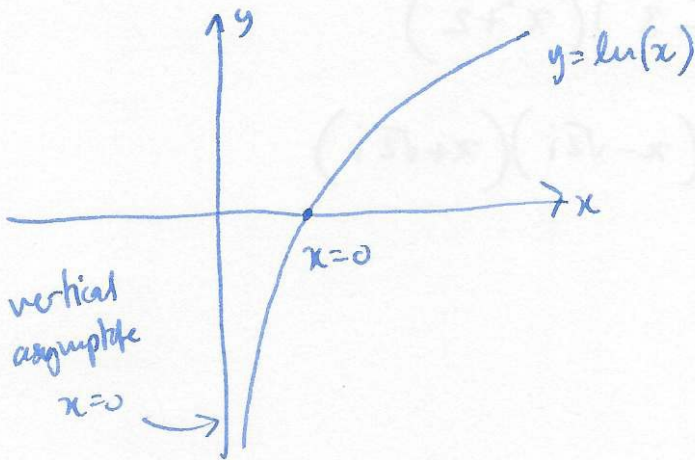
(b)  $zw$

$$\begin{aligned} (3 - 2i)(-2 + i) &= -6 + 3i + 4i + 2 \\ &= -4 + 7i \end{aligned}$$

(c)  $w/z$

$$\begin{aligned} \frac{(-2 + i)(3 + 2i)}{(3 - 2i)(3 + 2i)} &= \frac{-6 - 4i + 3i - 2}{9 + 4} \\ &= \frac{-8 - i}{13} \end{aligned}$$

- (4) (10 points) Sketch the graph of  $f(x) = \ln(x - 3) + 1$ , including any vertical or horizontal or vertical asymptotes the graph has, if any.



- (5) (10 points) Find all roots of the polynomial  $x^6 - x^4 - 6x^2$ , and state their multiplicities.

$$\begin{aligned}
 x^2(x^4 - x^2 - 6) &= x^2(x^2 - 3)(x^2 + 2) \\
 &= x^2(x - \sqrt{3})(x + \sqrt{3})(x - \sqrt{2}i)(x + \sqrt{2}i)
 \end{aligned}$$

| root         | multiplicity |
|--------------|--------------|
| 0            | 2            |
| $\sqrt{3}$   | 1            |
| $-\sqrt{3}$  | 1            |
| $\sqrt{2}i$  | 1            |
| $-\sqrt{2}i$ | 1            |

- (6) (10 points) Use the quadratic formula to find all roots of  $f(x) = 3x^2 + 4x + 6$ , expressing them in the form  $a + bi$ , where  $a$  and  $b$  are real numbers.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{16 - 4 \times 3 \times 6}}{6}$$

$$x = \frac{-4 \pm \sqrt{16 - 72}}{6}$$

$$x = \frac{-4 \pm \sqrt{-56}}{6}$$

$$= -\frac{2}{3} + \frac{\sqrt{56}i}{6}, -\frac{2}{3} - \frac{\sqrt{56}i}{6}$$

$$= -\frac{2}{3} + \frac{\sqrt{14}i}{3}, -\frac{2}{3} - \frac{\sqrt{14}i}{3}$$

(7) (10 points) Use the log rules to:

(a) Evaluate  $\log_4(16)$ .

2

(b) Expand  $\ln(x^2/\sqrt{y})$ .

$$= \ln(x^2) - \ln(\sqrt{y})$$

$$= 2\ln(x) - \frac{1}{2}\ln(y)$$

(c) Combine  $\log_2(x+1) - \frac{1}{3}\log_2(x+2)$  as a single logarithm.

$$\log_2(x+1) - \log_2(\sqrt[3]{x+2})$$

$$= \log_2\left(\frac{x+1}{\sqrt[3]{x+2}}\right)$$



(8) (10 points) Solve the following equations.

(a)  $e^{2x+3} = 4$

$$2x+3 = \ln(4)$$

$$2x = \ln(4) - 3$$

$$x = \frac{1}{2}(\ln(4) - 3) \quad \square$$

(b)  $\log_4(x+7) - \log_4(x-1) = 2$

$$\log_4\left(\frac{x+7}{x-1}\right) = 2$$

$$\frac{x+7}{x-1} = 4^2 = 16$$

$$x+7 = 16x-16$$

$$23 = 15x$$

$$x = \frac{23}{15}$$

(9) (10 points) You put \$500 in a bank account with 6% interest per year.

(a) If the interest is compounded monthly, how much will you have after 4 years?

$$P \left( 1 + \frac{r}{n} \right)^{nt} \quad 500 \left( 1 + \frac{0.06}{12} \right)^{12 \times 4} \approx 635.24$$

(b) If the interest is compounded continuously, how much will you have after 4 years?

$$P e^{rt} \quad 500 e^{0.06 \times 4} \approx 635.62$$

- (10) (10 points) You put \$500 in a bank account with 6% interest per year. If interest is compounded continuously, how long will it be before you have \$800?

$P e^{rt}$

$$500 e^{0.06t} = 800$$

$$e^{0.06t} = \frac{8}{5}$$

$$0.06t = \ln\left(\frac{8}{5}\right)$$

$$t = \frac{\ln\left(\frac{8}{5}\right)}{0.06} \approx 7.83 \text{ years}$$