Q1 $-8x^2+x-4 = -8\left(x^2-\frac{1}{8}x\right)-4$

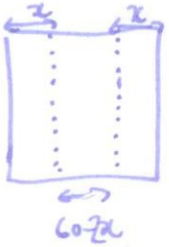
$= -8\left(\left(x-\frac{1}{16}\right)^2-\frac{1}{256}\right)-4$

$= -8\left(x^2-\frac{1}{8}x+\frac{1}{256}-\frac{1}{256}\right)-4$

$= -8\left(x-\frac{1}{16}\right)^2+\frac{1}{32}-4$

max value $\frac{1}{32}-4=-\frac{127}{32}$.

Q2 $V = (60-2x)x = (60x-2x^2)\times 10$



$= -120(x^2-30x)$

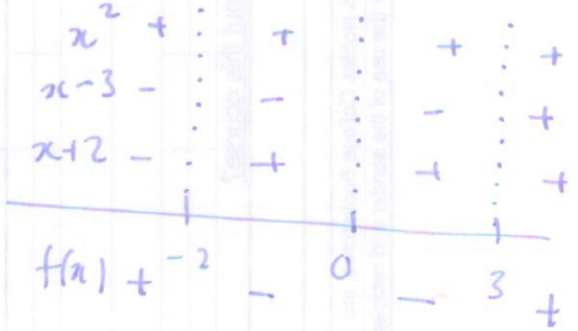
$= -120\left((x-15)^2-225\right)$

$= -120(x^2-30x+225-225)$

$= -120(x-15)^2+450\ 120\times 225$

max volume 27000 ~~450~~ cm^3

Q3 $x^4-x^3-6x^2 = x^2(x^2-x-6) = x^2(x-3)(x+2)$



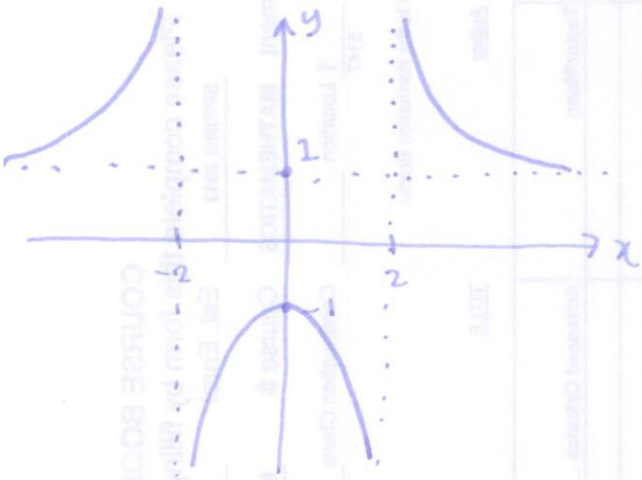
Q4 $f(x) = \dfrac{x^2+4}{x^2-4}$

x-intercepts : none

y-intercept : -1

vertical asymptotes : $x^2-4 = (x-2)(x+2)$ $\quad x = \pm 2$

horizontal asymptotes : $\dfrac{x^2}{x^2} = 1$



		-2		2	
x^2+4	+		+		+
$x+2$	−		+		+
$x-2$	−		−		+
$f(x)$	+	‖	−	‖	+

Q5 $(1+2i)(3i-1) = 3i - 1 - 6 - 2i = -7+i$

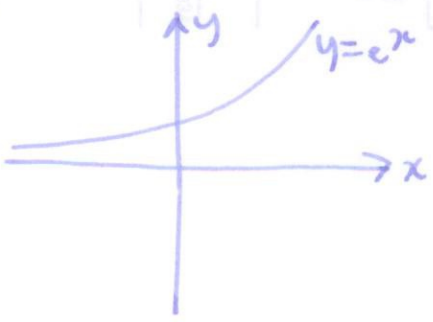
$$\frac{-7+i}{-3+2i}\,\frac{-3-2i}{-3-2i} = \frac{21+14i-3i+2}{9+4} = \frac{23+11i}{13} = \frac{23}{13} + \frac{11}{13}i$$

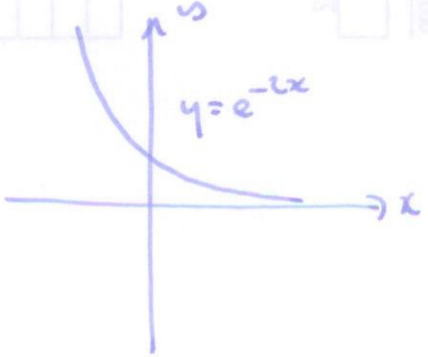
Q6 $x^4 - 2x^2 - 8 = (x^2)^2 - 2(x^2) - 8 = (x^2-4)(x^2+2)$

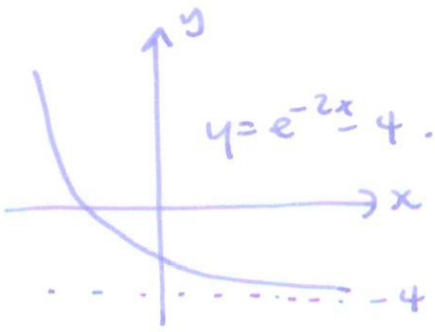
$= (x-2)(x+2)(x-\sqrt{2}i)(x+\sqrt{2}i)$

roots : $-2, +2, -\sqrt{2}i, +\sqrt{2}i$.

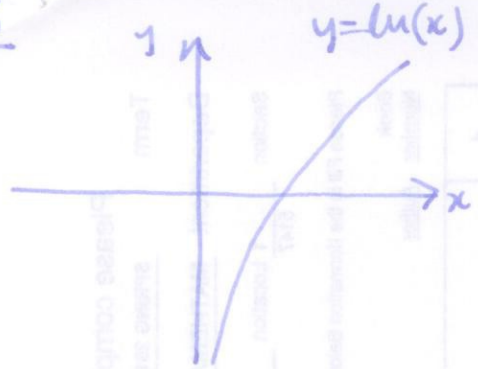
Q7

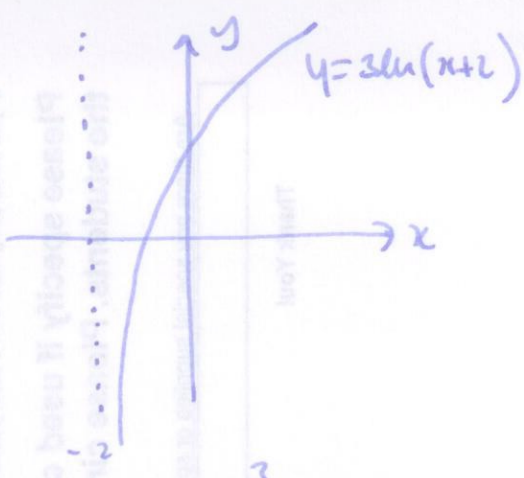






Q8





Q9 a) $P\left(1+\frac{r}{n}\right)^{rt}$ $\quad 200\left(1+\frac{0.04}{1}\right)^{3} = \$224.97$

b) Pe^{rt} $\quad 200\,e^{0.04\times 3} = \$225.50$

c) continuous

$200\,e^{0.04t} = 300$

$e^{0.04t} = \frac{3}{2}$

$0.04t = \ln(3/2)$

$t = \frac{\ln(3/2)}{0.04} = 10.14$ years

c) annual

$200(1+\frac{0.04}{1})^{t} = 300$

$1.04^{t} = \frac{3}{2}$

$\ln(1.04^{t}) = \ln\frac{3}{2}$

$t\ln(1.04) = \ln(3/2)$

$t = \frac{\ln(3/2)}{\ln(1.04)}$

$t \simeq 10.33$

Q10 a) $\log_4(1/2) = -1/2$

b) $\ln(\frac{1}{e}) = -1$

c) $\ln(a^3 b^4/\sqrt[3]{c}) = 3\ln(a) + 4\ln(b) - \frac{1}{3}\ln(c)$

d) $\ln\left(\sqrt{\frac{x+1}{x-1}}\right) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) = \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x-1)$.

e) $\ln\left(\sqrt{a\sqrt{b}}\right) = \frac{1}{2}\ln(a\sqrt{b}) = \frac{1}{2}\ln(a) + \frac{1}{2}\ln(\sqrt{b})$.

$= \frac{1}{2}\ln(a) + \frac{1}{4}\ln(b)$.

Q11

$f(x) = \ln\left(\frac{3x}{5}\right)$ $\qquad y = \ln\left(\frac{3x}{5}\right)$.

$e^y = \frac{3x}{5}$

$\frac{5e^y}{3} = x$

$f^{-1}(x) = \frac{5}{3}e^x$

Q12 a) $4^{2x+1} = 5^{3x}$ $\qquad \ln\left(4^{2x+1}\right) = \ln\left(5^{3x}\right)$

$(2x+1)\ln(4) = 3x\ln(5)$

$x \cdot 2\ln(4) - x \cdot 3\ln(5) = -\ln(4)$.

$x = \frac{-\ln(4)}{2\ln(4) - 3\ln(5)}$.

b) $\log_2(x-2) + \log_2(x+1) = 2$

$\log_2\left(\frac{(x-2)}{(x+1)}(x+1)\right) = 2$

$(x-2)(x+1) = 4$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ $\qquad x = 3, -2$ in fact only $x = 3$ works

c) $3\ln(5-x) = 2$

$\ln(5-x) = \frac{2}{3}$.

$5 - x = e^{2/3}$.

$x = 5 - e^{2/3}$.