Math 130 Precalculus Fall 14 Midterm 1a

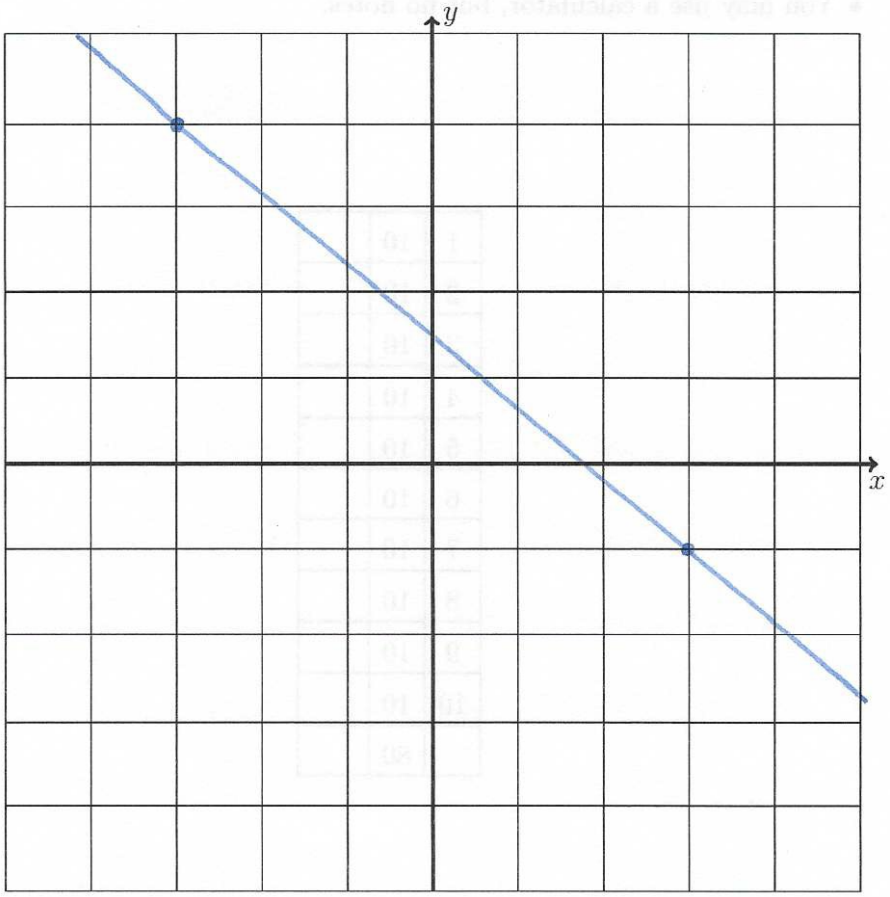
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-3, 4)$ and $(3, -1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



4

$$\text{slope} = \frac{-1-4}{3-(-3)} = \frac{-5}{6}$$

$$y - 4 = -\frac{5}{6}(x - (-3))$$

$$\boxed{y - 4 = -\frac{5}{6}(x+3)}$$

6

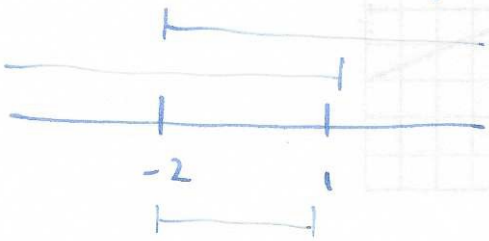
$$y = -\frac{5}{6}x - \frac{5}{2} + 4$$

$$y = -\frac{5}{6}x + \frac{3}{2}$$

(2) (10 points) Find the domain of the function $f(x) = \sqrt{x+2} + \sqrt{1-x}$.

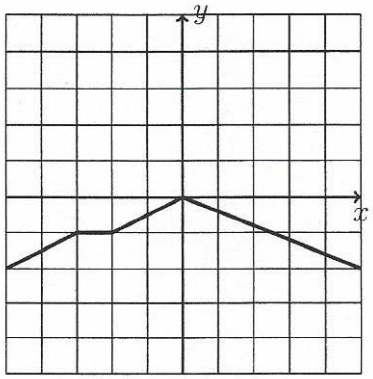
domain of $\sqrt{x+2}$ is $x \geq -2$ $[-2, \infty)$

$\sqrt{1-x}$ is $x \leq 1$ $(-\infty, 1]$



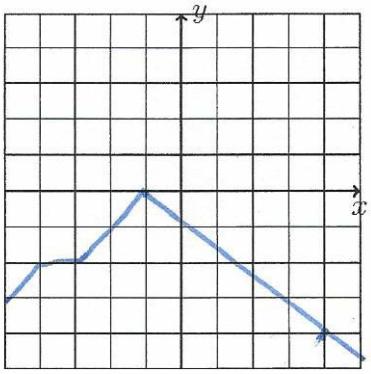
intersection is $[-2, 1]$

(3) (10 points) The graph of the function $f(x)$ is given below.



$f(x)$

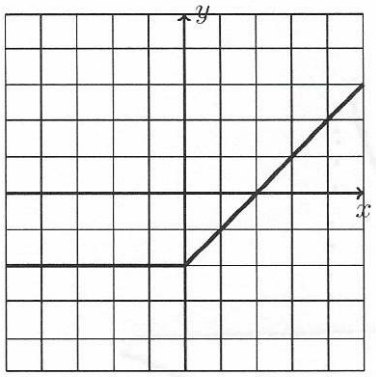
(a) Sketch the graph obtained from the graph of $f(x)$ by expanding it by a factor of 2 in the vertical direction, and then shifting it 1 unit to the left.



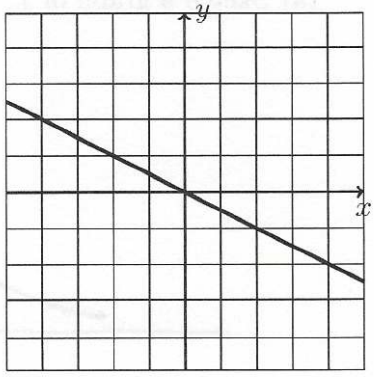
(b) Write down a formula for the function you have drawn above, expressed in terms of $f(x)$. Do not attempt to find a formula for $f(x)$ in terms of x.

$2f(x+1)$

(4) (10 points) The graphs of the functions $f(x)$ and $g(x)$ are shown below.

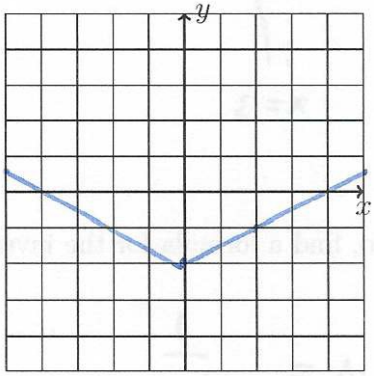


$f(x)$

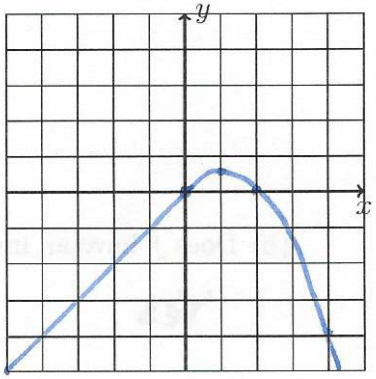


$g(x)$

(a) Sketch graphs of the following functions.



$f(x) + g(x)$



$f(x)g(x)$

(b) Find $(f \circ g)(4)$.

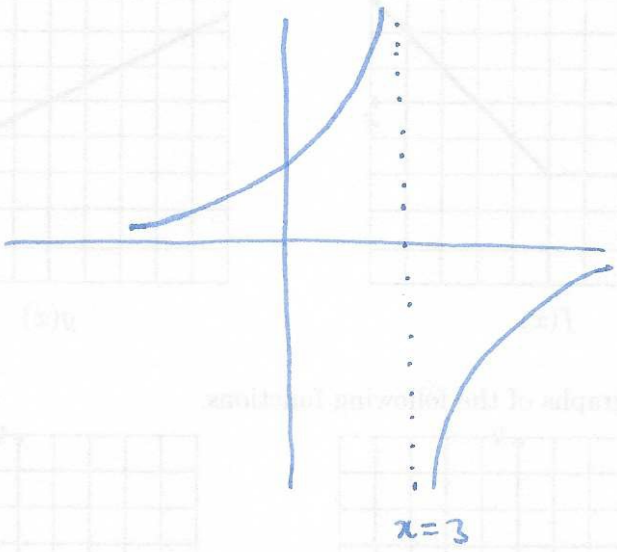
$f(g(4)) = f(-2) = -2$

(c) Does g have an inverse? If so, find $g^{-1}(-1)$.

Yes $\quad g^{-1}(-1) = -2$

(5) (10 points) Let $f(x) = \dfrac{1}{3-x}$.

(a) Sketch a graph of f.



(b) Does f have an inverse? If so, find a formula for the inverse.

Yes.

$$y = \frac{1}{3-x}$$

$$y(3-x) = 1$$

$$3y - xy = 1$$

$$3y - 1 = xy$$

$$x = \frac{3y-1}{y} = 3 - \frac{1}{y}$$

$$f^{-1}(x) = 3 - \frac{1}{x}$$

(6) (10 points) Use the method of completing the square to find the largest value of the function $1 + x - 2x^2$.

$$-2x^2 + x + 1$$

$$-2\left(x^2 - \frac{1}{2}x - \frac{1}{2}\right)$$

$$-2\left(\left(x - \frac{1}{4}\right)^2 - \frac{1}{16} - \frac{1}{2}\right) = \boxed{-2\left(x - \frac{1}{4}\right)^2 + \frac{9}{8}}$$

$$-2\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}\right)$$

Largest value of function is $\frac{9}{8}$

(7) (10 points) Find the values of x for which $x^2 + x < 12$.

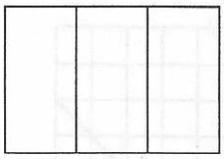
$x^2 + x - 12 < 0$

$(x-3)(x+4) < 0$

$(x-3)$	$-$	$-$	$+$
$(x+4)$	$-$	$+$	$+$
$(x-3)(x+4)$	$+$	$-$	$+$

answer: $(-4, 3)$

(8) (10 points) A farmer wishes to create three adjacent rectangular fields, as drawn below.



If the farmer has 200ft of fencing, what is the maximum area of the fields?

area $A = xy$

length $200 = 2x + 4y$

$100 = x + 2y$

$x = 100 - 2y$

$A = (100 - 2y)y$

$= 100y - 2y^2$

$-2(y^2 - 50y)$

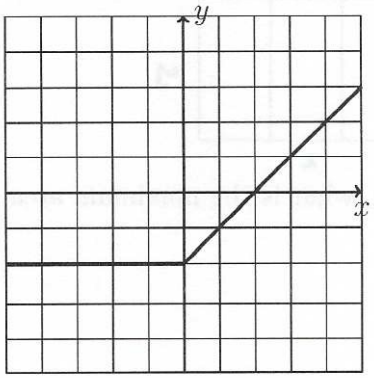
$-2\left((y-25)^2 - 625\right)$ $= -2(y-25)^2 + 1250$

$-2(y^2 - 50y + 25^2 - 625)$

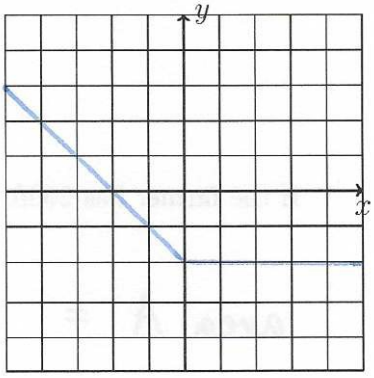
max area = $1250\ ft^2$

(9) (10 points) The graph of the function $f(x)$ is shown below.

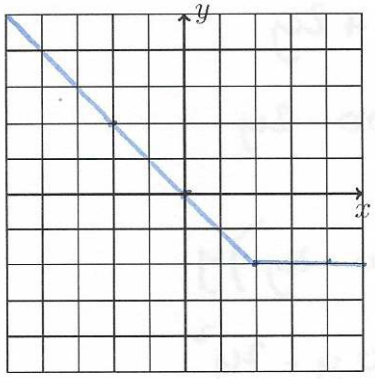
(a) Sketch the graphs of the other functions. Hint: do them in order.



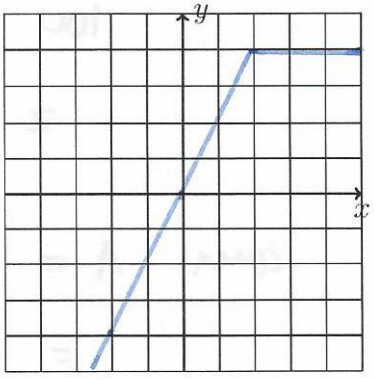
$f(x)$



$f(-x)$



$f(-x+2)$



$-2f(-x+2)$

(b) What is the average value of $f(x)$ between $x = -1$ and $x = 1$?

average value $= \dfrac{f(1)-f(-1)}{1-(-1)} = \dfrac{-1-(-2)}{2} = \dfrac{1}{2}$

(10) (10 points) Let $f(x) = 4x^2 - 2x + 3$.

(a) What value of x gives the minimum value of the function?

$4\left(x^2 - \frac{1}{2}x\right) + 3$

$4\left(\left(x - \frac{1}{4}\right)^2 - \frac{1}{16}\right) + 3 = 4\left(x - \frac{1}{4}\right)^2 + \frac{11}{4}$

$4\left(x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16}\right)$

value of x at min: $x = \frac{1}{4}$

(b) What is the minimum value of the function?

minimum value $\frac{11}{4}$