Sample midterm 1   Solutions

Q1 a) $x^2 - 2x - 15 = (x-5)(x+3)$

	-3		5	
	$+$	$-$	$+$	
$(x-5)$	$-$	$-$	$+$	
$(x+3)$	$-$	$+$	$+$	

domain $(-\infty, -3] \cup [5, \infty)$

b)

		-1		3	
$x+1$	$-$		$+$		$+$
$x-3$	$-$		$-$		$+$
$(x+1)(x-3)$	$+$		$-$		$+$

domain $(-\infty, -1] \cup (3, \infty)$

Q2 a) domain of f : $[-3, \infty)$

g : $\mathbb{R} \setminus \{\pm\sqrt{2}\} = (-\infty, \sqrt{2}) \cup (-\sqrt{2}, +\sqrt{2}) \cup (\sqrt{2}, \infty)$

domain of $f+g$: $[-3, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$.

b) $(f+g)(1) = f(1) + g(1) = \sqrt{4} + \frac{1}{1} = 3$.

c) $(f-g)(6) = f(6) - g(6) = \sqrt{9} + \frac{1}{2-36} = 3 - \frac{1}{32} = \frac{95}{32}$.

d) $(fg)(0) = f(0)g(0) = \sqrt{3} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$

e) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{\sqrt{4}}{1} = 2$

f) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2-x^2}\right) = \sqrt{\frac{1}{2-x^2} + 3} = \sqrt{\frac{8-3x^2}{2-x^2}}$

g) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{2-x^2}\right) = \frac{1}{2 - \left(\frac{1}{2-x^2}\right)^2} = \frac{(2-x^2)^2}{2(2-x^2)^2 - 1}$

$= \frac{(2-x^2)^2}{7 - 8x^2 + x^4}$

Q3 a) $f(x) = x^2 + 4x + 5$

$f(x) = (x+2)^2 + 1$ minimum value of f is 1.

$x^2 + 2x + 4 + 1$

b) $f(x) = 2x^2 + 4x + 4$

$= 2(x^2 + 2x) + 4$

$= 2\left[(x+1)^2 - 1\right] + 4$ $f(x) = 2(x+1)^2 + 2$

$= 2\left[x^2 + 2x + 1\right] + 4$ minimum value of f is 2.

Q4 a) $x^2 - 7x + 12 \geq 0$

$(x-3)(x-4) \geq 0$

	$(-\infty, 3)$	$(3, 4)$	$(4, \infty)$
$(x-3)$	$-$	$+$	$+$
$(x-4)$	$-$	$-$	$+$
$(x-3)(x-4)$	$+$	$-$	$+$

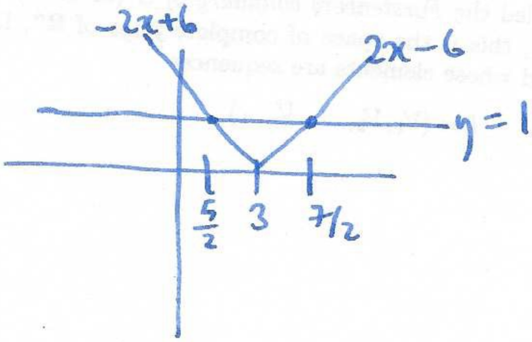
answer: $(-\infty, 3] \cup [4, \infty)$

b) $\dfrac{x+1}{x-4} > 2$ $\quad \dfrac{x+1}{x-4} - 2 > 0$ $\quad \dfrac{x+1-2(x-4)}{x-4} > 0$ $\quad \dfrac{-x+7}{x-4} > 0$

	$(-\infty, 4)$	$(4, 7)$	$(7, \infty)$
$-x+7$	$+$	$+$	$-$
$x-4$	$-$	$+$	$+$
	$-$	$+$	$-$

answer: $(4, 7)$.

c)

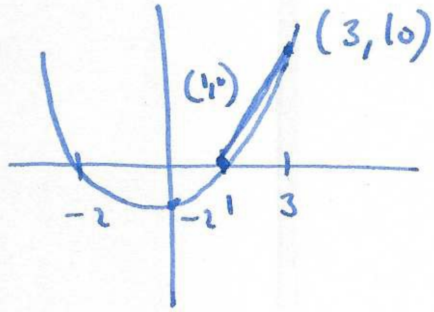


$2x - 6 = 1 \Rightarrow x = 7/2$

$-2x + 6 = 1 \Rightarrow x = 5/2$

answer: $(-\infty, \frac{5}{2}) \cup (7/2, \infty)$

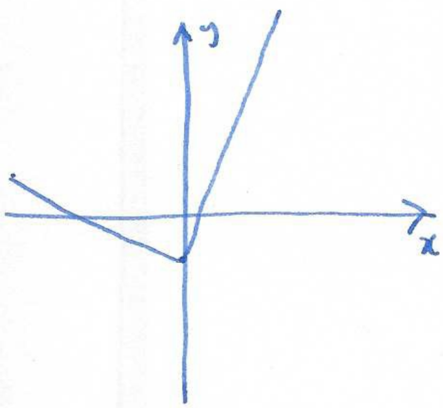
Q5 $f(x) = x^2 + x - 2 = (x+2)(x-1)$



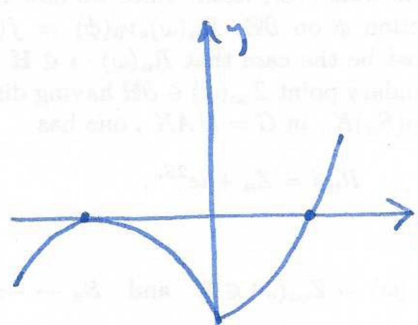
average rate of change from $x=1$ to $x=3$:

$$\frac{10-0}{3-1} = 5.$$

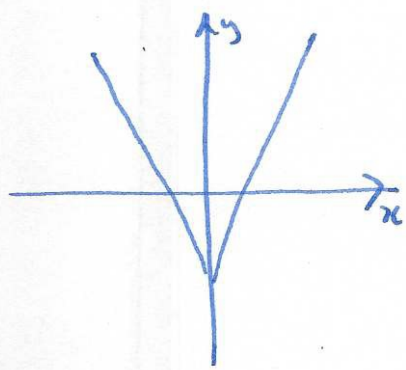
Q6 a)



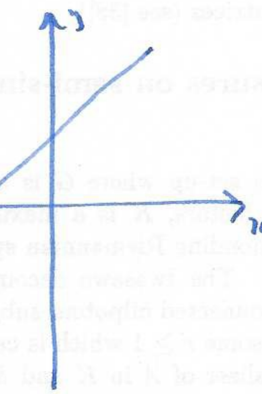
b)



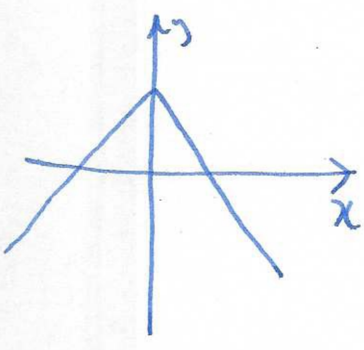
c)



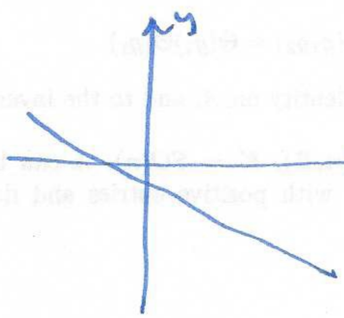
d)



e)



f)



Q7 a) $f(g(2)) = f(-1) = 1$

b) $(g \circ f)(-1) = g(f(-1)) = g(1) = -2$

c) $f(g(g(1))) = f(g(-2)) = f(-1) = 1$

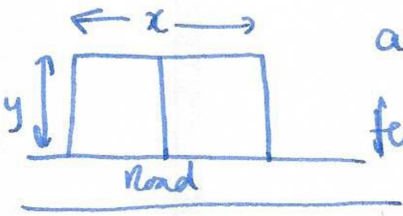
④

d) f has an inverse as it is one-to-one, it passes the horizontal line test. $f^{-1}(3) = 3$

e) g does not have an inverse, there is a horizontal line which hits the graph twice.

f) f : neither odd nor even g : even

Q8



area: $A = xy$
fence: $x + 3y = 600$ $\Big\}$ $A = (600 - 3y)y$

$$A = 600y - 3y^2 = -3y^2 + 600y = -3(y^2 - 200y)$$
$$= -3\left((y-100)^2 - 10000\right)$$
$$-3\left(y^2 - 200y + 10000 - 10000\right)$$

max value of A when $y = 100$
$x = 300$ $A = 30000$