Math 130 Precalculus Fall 2014 Sample final

(1) Find the domain of the following functions.

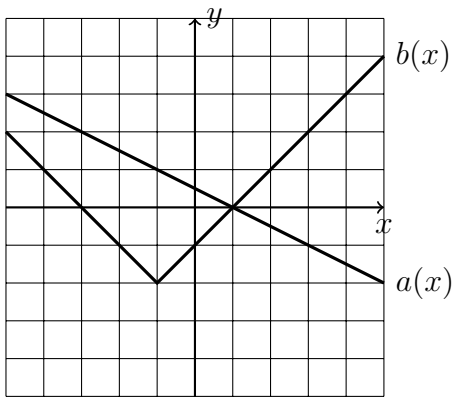
(a) $\dfrac{2x-1}{5-3x}$

(b) $\ln\left(\dfrac{2x-1}{5-3x}\right)$

(c) $\dfrac{y^2-1}{y^3-4y^2-12y}$

(d) $\sqrt{(x-1)(x+2)}$

(2) The graphs of two functions $a(x)$ and $b(x)$ are shown below. Find:



(a) $a(b(-1))$

(b) $b(b(a(1)))$

(c) Sketch $\frac{1}{2}b(x)-2$

(d) Sketch $b(x/2)$

(e) Sketch $a(x)+b(x)$

(f) Sketch $a(x)b(x)$

(3) Find formulas for the inverse function of the following functions.

(a) $\dfrac{3x-2}{2-5x}$

(b) $\sqrt{3x-2}$

(c) $e^{2x+1}-1$

(d) $2\ln(x-3)-1$

(4) Complete the square for $-3x^2+2x-3$.

(5) Find the maximum value of $\cos\theta - 3\cos^2\theta$ by completing the square.

(6) I wish to build a bookshelf with four sides and three horizontal shelves. If I have 24 feet of planks, what dimensions of the sides maximizes the front facing area of the bookshelf?

(7) Factor the following polynomials, and list their roots, with multiplicities.
   (a) $x^4 + x^2 - 2$

   (b) $8x^6 + 10x^4 - 3x^2$

(8) Find a polynomial of degree 4 with real cooefficients and roots $1+i$ and $2-i$.

(9) Sketch the graph of the functions

   (a) $(x-3)(4x-1)(x+4)$.

   (b) $\dfrac{8-2x^2}{x^2+x-12}$

(10) You leave \$250 in my bank account at 5% interest per year.

   (a) How much will you have after 3 years, with interest compounded monthly?

   (b) How much will you have after 6 months, with interest compounded continuously?

   (c) How long will it be before you have \$400?

(11) Solve:

   (a) $\ln(x-3) = \ln(x+3) - 4$

   (b) $\ln(2x-1) = \ln(3x+2) - 4$

   (c) $4e^{4x} + 5x^{2x} - 6 = 0$

   (d) $e^x - 3e^{-x} = -2$

   (e) $\cos(x) = -\sqrt{2}/2$

   (f) $3\cos^2 x + 5\cos x = 2$

   (g) $3\cos^2 x + 5\sin x = 1$

   (h) $\sin x = 2\cos x + 1$

(12) Find the point on the unit circle which is the terminal point for $t = 29\pi/6$.

(13) Find the exact value of:

(a) $\cos(-21\pi/6)$

(b) $\cot(21\pi/4)$

(c) $\csc(-13\pi/3)$

(d) $\cos^{-1}(-\sqrt{3}/2)$

(e) $\sin^{-1}(\sin(27\pi/7))$

(f) $\cos(\cot^{-1}(8/3))$

(g) $\cos(5\pi/12)$

(h) $\cos(3\pi/20)\cos(\pi/10) - \sin(3\pi/20)\sin(\pi/10)$

(14) If $\csc(t) = -9/4$ and t is in quadrant 3, find the values of the other trig functions at t.

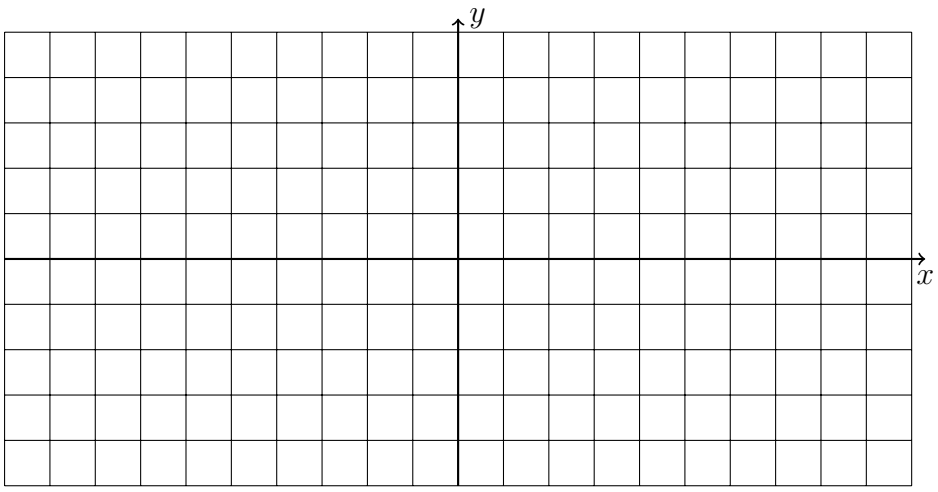
(15) Decide if the following functions are even, odd or neither:

(a) $\cos(x)\sin(x)$

(b) $(e^x + e^{-x})\tan x$

(c) $\sin(x) + \cos(x)$

(16) Find the amplitude, frequency and phase shift for $y = -2\sin(x/2 - \pi/6)$, and draw a careful graph of the function.



(17) The angle a pendulum makes with a vertical line changes according to simple harmonic motion. If the pendulum is released from angle $x = \pi/3$ and takes $1/4$ of a second to reach the vertical position, write down an equation describing the motion of the pendulum.

(18) Verify the following trig identities:

(a) $\tan x + \cot x = \sec x \csc x$

(b) $\dfrac{1}{1 - \sin x} - \dfrac{1}{1 + \sin x} = 2 \tan x \sec x$

(c) $\dfrac{\sin^2 x}{\cos^2 x + 3 \cos x + 2} = \dfrac{1 - \cos x}{2 + \cos x}$

Useful formulas

Fundamental identities

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x)$$

$$\tan(-x) = -\tan(x)$$

Addition and subtraction formula

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$

$$\tan(A-B) = \frac{\tan(A)-\tan(B)}{1+\tan(A)\tan(B)}$$

Special cases

$$\sin(x+\frac{\pi}{2}) = \cos(x)$$

$$\sin(x-\frac{\pi}{2}) = -\cos(x)$$

$$\cos(x+\frac{\pi}{2}) = -\sin(x)$$

$$\cos(x-\frac{\pi}{2}) = \sin(x)$$

Inverses

Function	Domain	Range
$\sin^{-1} x$	$[-1,1]$	$[-\frac{\pi}{2},\frac{\pi}{2}]$
$\cos^{-1} x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1} x$	(∞,∞)	$(-\frac{\pi}{2},\frac{\pi}{2})$

Double angle

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Half angle

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

$$= \frac{1-\cos x}{\sin x}$$

Interest rates

$A(t)$ amount after t years
P principal
r interest rate per year
n number of times compounded per year
t number of years

Discrete compounding

$A(t) = P(1+\frac{r}{n})^{nt}$

Continuous compounding

$A(t) = Pe^{rt}$