Sample final Solutions

Q1 a) $5-3x=0 \Rightarrow x=\frac{5}{3}$

domain: $\boxed{\left(-\infty,\frac{5}{3}\right)\cup\left(\frac{5}{3},\infty\right)}$

b) need $\dfrac{2x-1}{5-3x}>0$

		$\frac{1}{2}$		$\frac{5}{3}$	
$2x-1$	$-$		$+$		$+$
$5-3x$	$+$		$+$		$-$
$\frac{2x-1}{5-3x}$	$-$		$+$		$-$

domain $\boxed{(1/2,\ 5/3)}$

c) $y^3+4y^2-12y = y(y^2+4y-12) = y(y+6)(y-2)$

domain $\boxed{(-\infty,-6)\cup(-6,0)\cup(0,2)\cup(2,\infty)}$

d) $\sqrt{(x-1)(x+2)}$ need $(x-1)(x+2)>0$

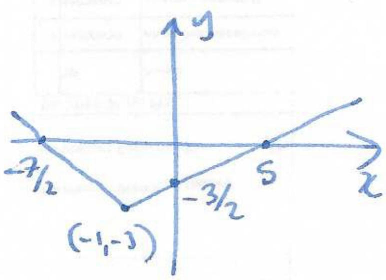
		-2		1	
$x-1$	$-$		$-$		$+$
$x+2$	$-$		$+$		$+$
$(x-1)(x+2)$	$+$		$-$		$+$

domain $\boxed{(-\infty,-2)\cup(1,\infty)}$

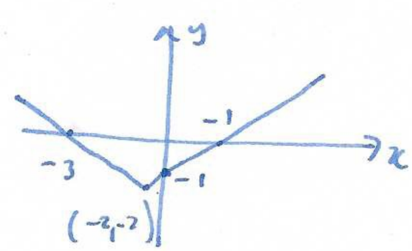
Q2 a) $b(-1)=-2$ $\quad a(b(-1))=a(-2)=\boxed{-3/2}$

b) $a(1)=0$ $\quad b(0)=-1$ $\quad b(-1)=\boxed{-2}$

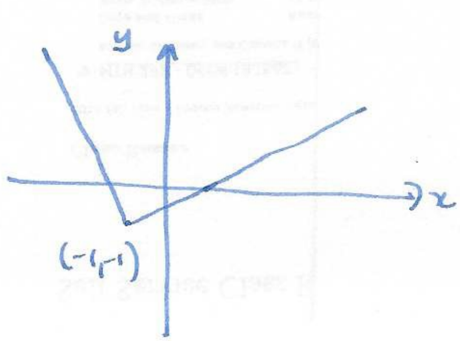
c)



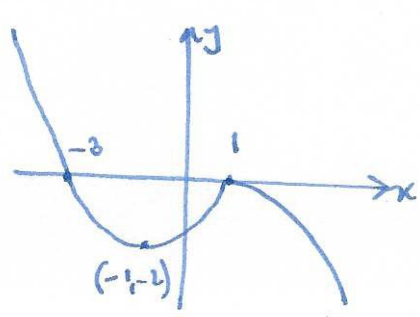
d)



e)



f)



Q3 a) $y = \frac{3x-2}{2-5x}$ $\quad 2y - 5xy = 3x - 2$ $\quad 2y + 2 = 3x + 5xy$

$x(3+5y) = 2y+2$ $\quad x = \frac{2y+2}{3+5y}$ $\quad \boxed{f^{-1}(x) = \frac{2x+2}{3+5x}}$.

b) $y = \sqrt{3x-2}$ $\quad y^2 = 3x-2$ $\quad y^2+2 = 3x$ $\quad \frac{1}{3}(y^2+2) = x$

$\boxed{f^{-1}(x) = \frac{1}{3}(x^2+2)}$

c) $y = e^{2x+1} - 1$ $\quad y+1 = e^{2x+1}$ $\quad \ln(y+1) = 2x+1$

$x = \frac{\ln(y+1) - 1}{2}$ $\quad \boxed{f^{-1}(x) = \frac{1}{2}(\ln(y+1) - 1)}$

d) $y = 2\ln(x-3) - 1$ $\quad y+1 = 2\ln(x-3)$ $\quad \frac{1}{2}(y+1) = \ln(x-3)$

$e^{(y+1)/2} = x - 3$ $\quad x = e^{(y+1)/2} + 3$ $\quad \boxed{f^{-1}(x) = e^{(y+1)/2} + 3}$

Q4 $-3\left(x^2 - \frac{2}{3}x + 1\right)$

$-3\left((x-\frac{1}{3})^2 + \frac{8}{9}\right) = \boxed{-3\left(x-\frac{1}{3}\right)^2 - \frac{8}{3}}$.

$-3\left(x^2 - \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} + 1\right)$

Q5 $-3\cos^2\theta + \cos\theta$

$-3\left(\cos^2\theta - \frac{1}{3}\cos\theta\right)$

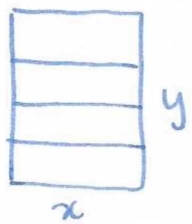
$-3\left(\left(\cos\theta - \frac{1}{6}\right)^2 - \frac{1}{36}\right)$

$$= -3\left(\cancel{2}\cos^2\theta - \frac{2}{6}\cos\theta + \frac{1}{36} - \frac{1}{36}\right)$$

$$= -3\left(\cos\theta - \frac{1}{6}\right)^2 + \frac{1}{12}$$

max value when $\cos\theta = 1/6$
max value is $\frac{1}{12}$.

Q6



$5x + 2y = 24 \} \quad y = 12 - \frac{5}{2}x$

$A = xy = \quad x\left(12 - \frac{5}{2}x\right)$

Max : $-\frac{5}{2}x^2 + 12x$

$$-\frac{5}{2}\left(x^2 - \frac{24}{5}x\right)$$

$$-\frac{5}{2}\left(\left(x - \frac{12}{5}\right)^2 - \frac{576}{25}\right) = -\frac{5}{2}\left(x - \frac{12}{5}\right)^2 + \frac{288}{5}$$

$$-\frac{5}{2}\left(x^2 - \frac{24}{5}x + \frac{576}{25} - \frac{576}{25}\right)$$

↑ maximum occurs when
$x = \frac{12}{5}$, $y = 6$

Q7

a) $x^4 + x^2 - 2 = \underbrace{(x^2+2)}_{x = \pm\sqrt{2}i}\underbrace{(x^2-1)}_{x = \pm 1} = (x - \sqrt{2}i)(x + \sqrt{2}i)(x-1)(x+1)$

roots:	$\sqrt{2}i$	$-\sqrt{2}i$	$+1$	-1
multiplicity:	1	1	1	1

b) $x^2(8x^4 + 10x^2 - 3) = x^2(4x^2 - 2)(2x^2 + 3)$

$$= 8x^2\left(x^2 - \frac{1}{2}\right)\left(x^2 + 3/2\right)$$

$$= 8x^2\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)\left(x + \sqrt{3/2}\,i\right)\left(x - \sqrt{3/2}\,i\right)$$

roots: $0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}i, -\sqrt{3/2}\,i$

multiplicity: 2 1 1 1 1

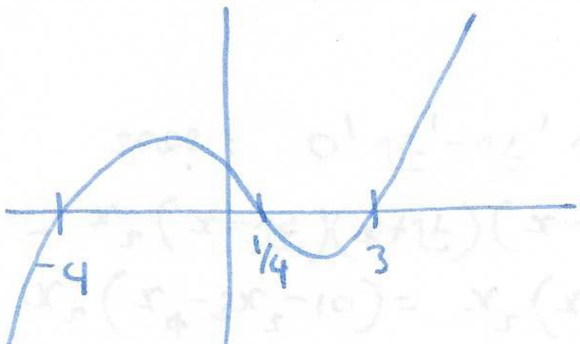
Q8 $(x-(1+i))(x-(1-i))(x-(2-i))(x-(2+i))$

$(x^2-2x+2)(x^2-4x+5)$

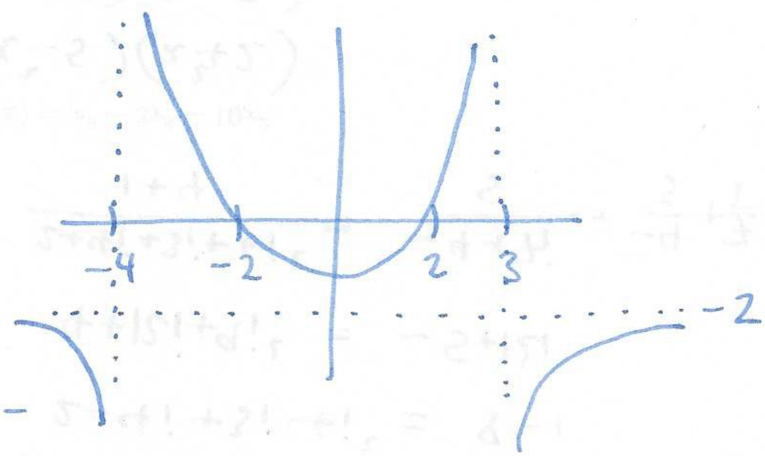
$$\begin{aligned} &x^4-4x^3+5x^2\\ &\quad -2x^3+8x^2-10x\\ &\quad\quad +2x^2-8x+10 \end{aligned}$$

$$\boxed{x^4-6x^3+15x^2-18x+10}$$

Q9 a)



b) $\dfrac{8-2x^2}{x^2+x-12} = \dfrac{-2(x+2)(x-2)}{(x-3)(x+4)}$



	$x<-4$	$-4<x<-2$	$-2<x<2$	$2<x<3$	$x>3$
$-2(x+2)$	+	+	−	−	−
$(x-2)$	−	−	−	+	+
$(x-3)$	−	−	−	−	+
$(x+4)$	−	+	+	+	+
$f(x)$	−	+	−	+	−

Q10 a) $P(1+\frac{r}{n})^{nt}$ $\quad 250\left(1+\frac{0.05}{12}\right)^{12\times 3} = \boxed{290.37}$

b) Pe^{rt} $\quad 250e^{0.05\times\frac{1}{2}} = \boxed{256.33}$

c) $250e^{0.05t} = 400$

$$e^{0.05t} = \frac{400}{250}$$

$$0.05t = \ln\left(\frac{400}{250}\right) \qquad t = 20\ln\left(\frac{8}{5}\right) \approx \boxed{9.4 \text{ years}}.$$

Q11 a) $\ln(x-3) = \ln(x+3) - 4$

$$\ln\left(\frac{x-3}{x+3}\right) = -4$$

$$\frac{x-3}{x+3} = e^{-4}$$

$$x - 3 = e^{-4}x + 3e^{-4}$$

$$x\left(1 - e^{-4}\right) = 3e^{-4} + 3$$

$$\boxed{x = \frac{3e^{-4}+3}{1-e^{-4}}}$$

b) $\ln\left(\frac{2x-1}{3x+2}\right) = -4$

$$\frac{2x-1}{3x+2} = e^{-4}$$

$$2x - 1 = 3e^{-4}x + 2e^{-4}$$

$$x\left(2 - 3e^{-4}\right) = 2e^{-4} + 1$$

$$\boxed{x = \frac{2e^{-4}+1}{2-3e^{-4}}}$$

c) $(4e^{2x}-3)(e^{2x}+2) = 0$

$e^{2x} = \frac{3}{4}$ $\qquad$ $e^{2x} = -2$
no solutions

$2x = \ln(3/4)$

$\boxed{x = \frac{1}{2}\ln(3/4)}$

d) $e^{x} + 2 - 3e^{-x} = 0$

$e^{2x} + 2e^{x} - 3 = 0$

$(e^{x}+3)(e^{x}-1) = 0$

$e^{x} = -3$
no solution

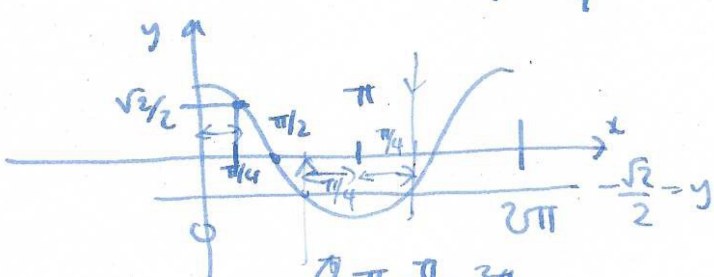
$e^{x} = 1$

$\boxed{x = 0}$

$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$.

e) $\cos(x) = -\frac{\sqrt{2}}{2}$

we: $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$



$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

solutions

$$\boxed{\begin{matrix} \frac{3\pi}{4} + 2\pi n \\ \frac{5\pi}{4} + 2\pi n \end{matrix} \quad n \in \mathbb{Z}}$$

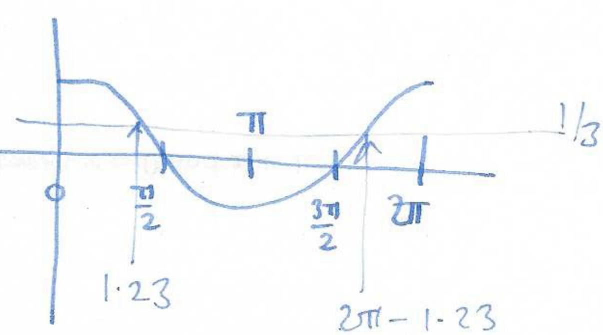
f) $3\cos^2 x + 5\cos x - 2 = 0$

$(3\cos x - 1)(\cos x + 2) = 0$

$\cos x = -2$
no solutions

$\cos x = \frac{1}{3}$

$x = \cos^{-1}(1/3) \approx 1.23$.



solutions: $\boxed{\begin{matrix} 1.23 + 2\pi n \\ -1.23 + 2\pi n \end{matrix} \quad n \in \mathbb{Z}}$

g) $3\cos^2 x + 5\sin x - 1 = 0$

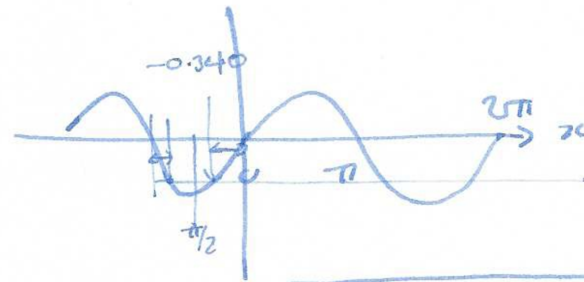
$3(1-\sin^2 x) + 5\sin x - 1 = 0$

$-3\sin^2 x + 5\sin x + 2 = 0$

$3\sin^2 x - 5\sin x - 2 = 0$

$(3\sin x + 1)(\sin x - 2) = 0$

$\sin x = -\frac{1}{3}$. $\quad\quad \sin x = 2$ no solutions



$\sin^{-1}(-1/3) \approx \boxed{\begin{array}{l} -0.340 + 2\pi n \\ -\pi + 0.340 + 2\pi n \end{array} \quad n\in\mathbb{Z}}$

h) $\sin x - 2\cos x = 1$

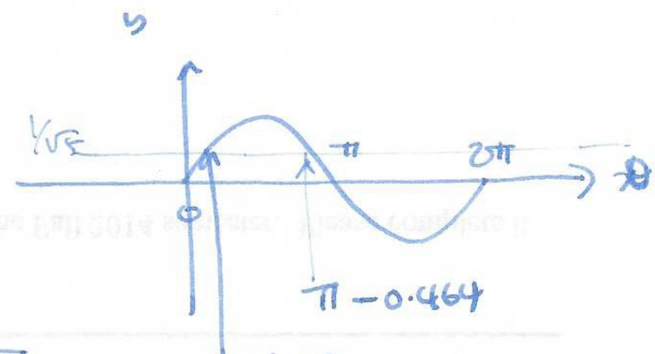
$A\sin(x+B) = A\sin x\cos B + A\cos x\sin B$.

$\left.\begin{array}{l} A\cos B = 1 \\ A\sin B = -2 \end{array}\right\}$ $\quad \tan B = -2 \quad\quad B = \tan^{-1}(-2) \approx -1.11$

$A^2\underbrace{(\sin^2 A + \cos^2 B)}_{1} = 4+1 \quad\quad A = \sqrt{5}$

$\sqrt{5}\sin(x - 1.11) = 1$

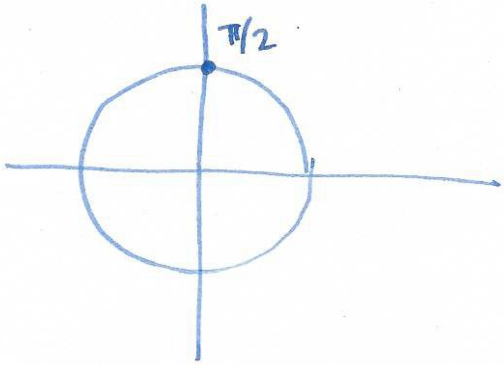
$\sin\underbrace{(x-1.11)}_{\theta} = 1/\sqrt{5}$



$x = \theta + 1.11 = \boxed{\begin{array}{l} 1.574 + 2\pi n \\ \pi + 0.646 + 2\pi n \end{array} \quad n\in\mathbb{Z}}$

$\theta = \sin^{-1}(1/\sqrt{5}) \approx 0.464$

Q12

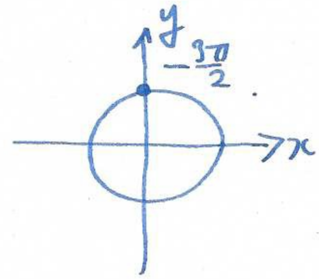


$$\frac{29\pi}{6} = 4\pi + \frac{3\pi}{6} = 4\pi + \frac{\pi}{2}$$

terminal point $\boxed{(0,1)}$

Q13 a) $\cos\left(-\frac{21\pi}{6}\right)$

$$-\frac{21\pi}{6} = -3\pi - \frac{3\pi}{6} = -3\pi - \frac{\pi}{2}$$



so $\cos\left(-\frac{21\pi}{6}\right) = 0$

b) $\cot\left(\frac{21\pi}{4}\right)$ $\qquad \frac{21\pi}{4} = 5\pi + \frac{\pi}{4}$

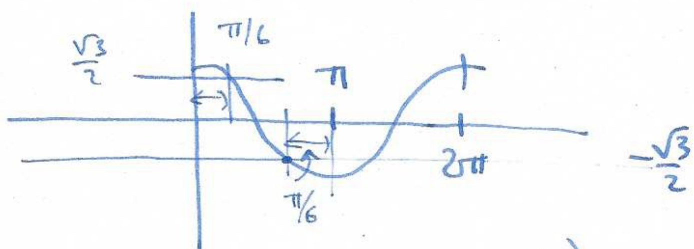
$= \cot\left(\frac{\pi}{4}\right) = \boxed{1}$

c) $\operatorname{cosec}\left(-\frac{13\pi}{3}\right) = \frac{1}{\sin\left(-\frac{13\pi}{3}\right)}$ $\qquad -\frac{13\pi}{3} = -4\pi - \frac{\pi}{3}.$

$\sin\left(-\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

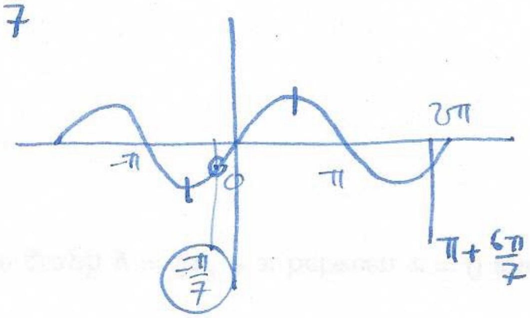
d) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

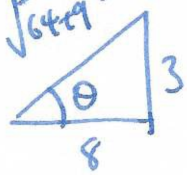


so $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

e) $\frac{27\pi}{7} = 3\pi + \frac{6\pi}{7} = 4\pi - \frac{\pi}{7}$



$\sin^{-1}\left(\sin\left(\frac{27\pi}{7}\right)\right) = -\frac{\pi}{7}.$

f) 

$$\cos\theta = \boxed{\frac{8}{\sqrt{73}}}$$

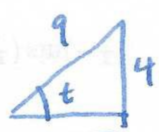
g) $$\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{6}\right) = \underbrace{\cos\left(\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}\underbrace{\cos\left(\frac{\pi}{6}\right)}_{\frac{\sqrt{3}}{2}} - \underbrace{\sin\left(\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}\underbrace{\sin\left(\frac{\pi}{6}\right)}_{\frac{1}{2}}$$

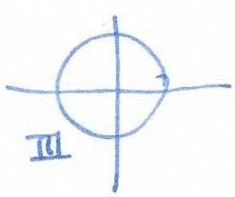
$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

h) $$= \cos\left(\frac{3\pi}{20}+\frac{\pi}{10}\right) = \cos\left(\frac{5\pi}{20}\right) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$$

Q14 $$\operatorname{cosec}(t) = -9/4 = \frac{1}{\sin(t)}$$



$$\sqrt{81-16} = \sqrt{65}$$



$$\cos(t) = \frac{\sqrt{65}}{9} \qquad \sec(t) = \frac{9}{\sqrt{65}}$$

$$\tan(t) = \frac{4}{\sqrt{65}} \qquad \cot(t) = \frac{\sqrt{65}}{4}.$$

Q15 a) $\cos(-x)\sin(-x) = -\cos(x)\sin(x)$ $\boxed{\text{odd}}$

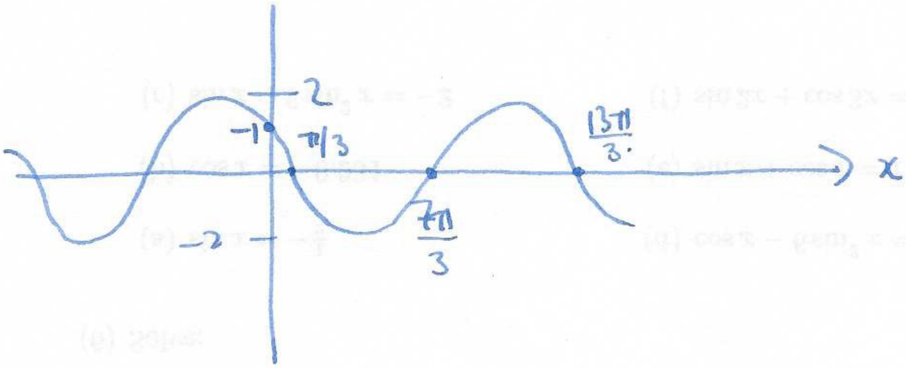
b) $(e^{-x}+e^{x})\tan(-x) = -(e^{x}-e^{-x})\tan(x)$ $\boxed{\text{odd}}$

c) $\sin(-x)+\cos(-x) = -\sin(x)+\cos(x)$ $\boxed{\text{neither}}$

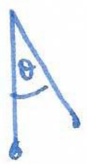
Q16 $y = -2\sin\left(\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right)$ amplitude 2

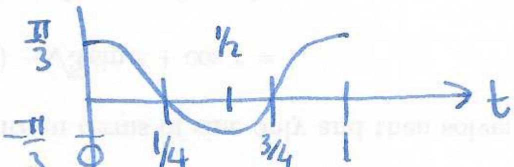
period $= \frac{2\pi}{k} = 4\pi$ frequency $= \frac{1}{\text{period}} = \frac{1}{4\pi}$.

phase shift $= \pi/3$.



Q17





period = 1 second

amplitude $= \frac{\pi}{3}$

$k = 2\pi$

phase shift $= 0$

$\theta = \frac{\pi}{3}\cos(2\pi t)$.

Q18 a) $\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \sec x \operatorname{cosec} x$.

b) $\frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{1+\sin x - 1 + \sin x}{1-\sin^2 x} = \frac{2\sin x}{\cos^2 x} = 2\tan x \sec x$

c) $\frac{\sin^2 x}{\cos^2 x + 3\cos x + 2} = \frac{1-\cos^2 x}{(\cos x + 2)(\cos x + 1)} = \frac{(1-\cos x)(1+\cos x)}{(\cos x+2)(\cos x+1)} = \frac{1-\cos x}{\cos x + 2}$.