Math 130 Precalculus Fall14, FINAL a

Name: Solutions

- No notes
- Cellphones must be switched OFF.
- You MUST EXPLAIN your answers and MUST show all your work

1	12	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
	100	

(1) (**12 points**) Consider the function

$$f(x) = \frac{x-3}{x-2}$$

(a) (2 points) Find the maximal domain for this ~~formula~~ function

$x \neq 2$ $\quad (-\infty, 2) \cup (2, \infty)$

(b) (2 points) Find the x- and y-intercepts.

y-intercept: $f(0) = \frac{3}{2}$

x-intercepts: solve $f(x) = 0$ $\quad x = 3$.

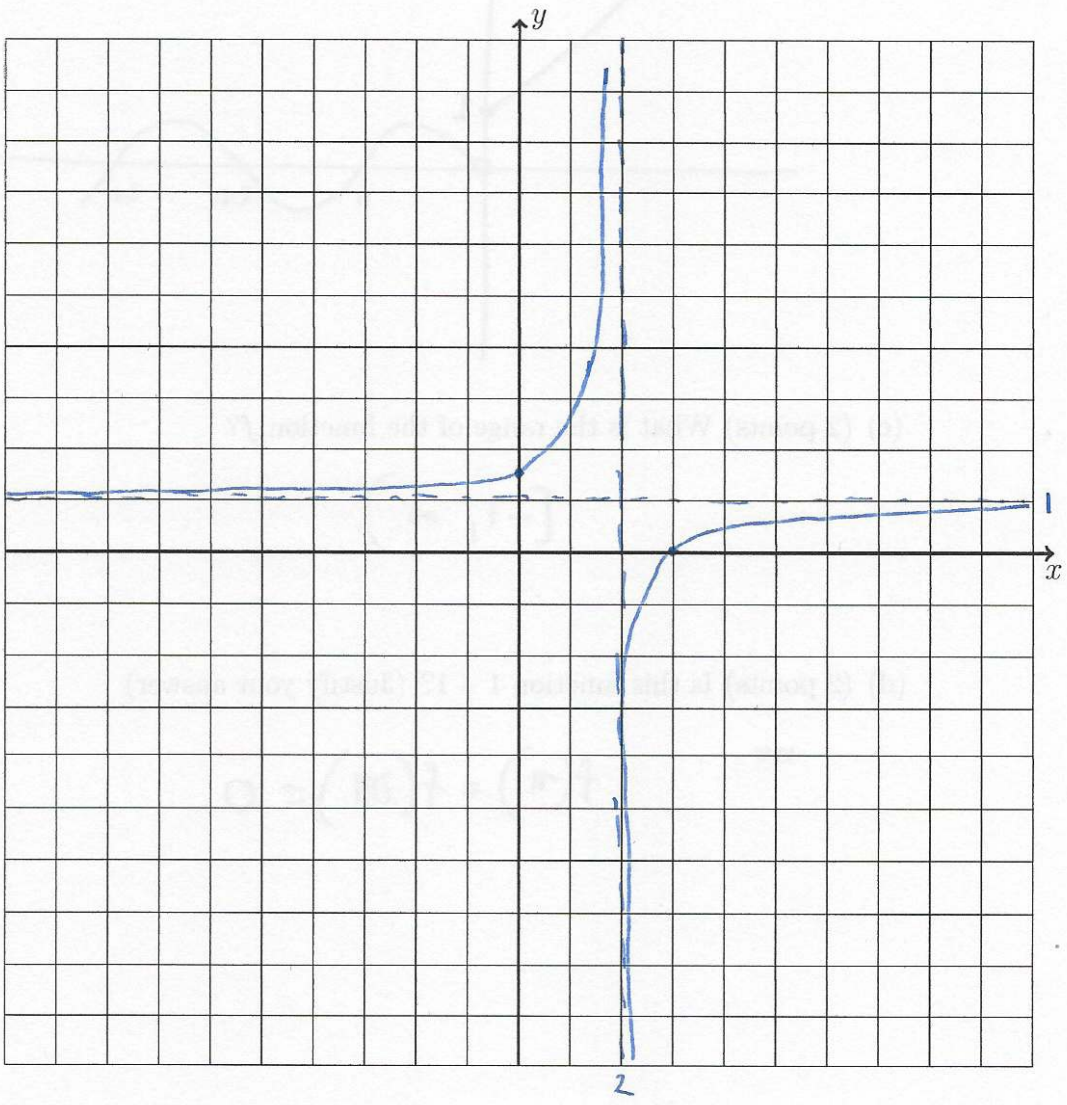
(c) (2 points) Examine how the function behaves when $x \to \pm\infty$

$$\frac{x-3}{x-2} \sim \frac{x}{x} = 1 .$$

(d) (2 points) Find equations of all the vertical asymptotes (if any).

$x = 2$

(e) (4 points) Based on all this information, sketch the graph of this function and find its range.



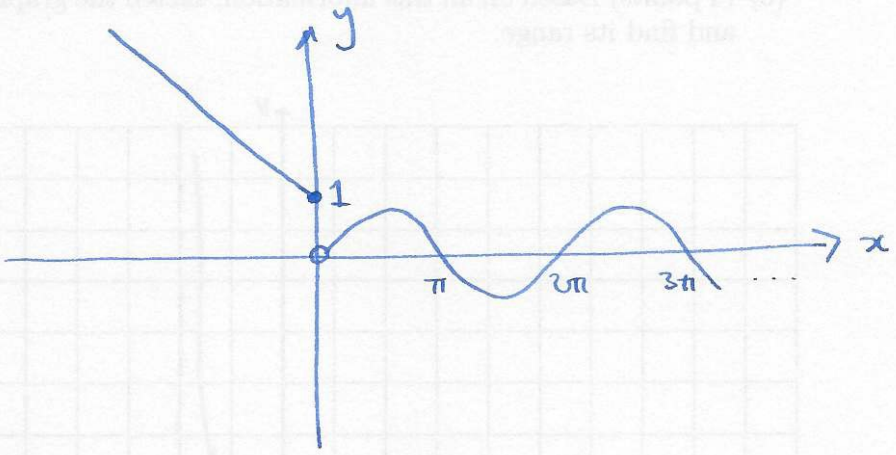
(2) (**8 points**) Consider the function

$$f(x) = \begin{cases} \sin x & x > 0 \\ -x + 1 & x \leq 0 \end{cases}$$

(a) (2 points) Find the values of $f(0)$, $f(-\pi)$ and $f(\pi)$.

$f(0) = 1 \quad f(-\pi) = \pi + 1 \quad f(\pi) = \sin(\pi) = 0$

(b) (2 points) Sketch the graph of the function f, indicating the x- and y-intercepts.



(c) (2 points) What is the range of the function f?

$[-1, \infty)$

(d) (2 points) Is this function $1-1$? (Justify your answer)

no.

$f(\pi) = f(2\pi) = 0$

(3) (**8 points**) A quadratic function is given $f(x) = 2x^2 + x - 1$.

(a) (2 points) Express the quadratic function in the standard form

$$f(x) = a(x-h)^2 + k$$

$2\left(x^2 + \frac{1}{2}x - \frac{1}{2}\right)$

$2\left(\left(x+\frac{1}{4}\right)^2 - \frac{1}{16} - \frac{1}{2}\right) = 2\left(x+\frac{1}{4}\right)^2 - \frac{1}{8} - 1$

$2\left(x^2 + \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}\right) = 2\left(x+\frac{1}{4}\right)^2 - \frac{9}{8}$

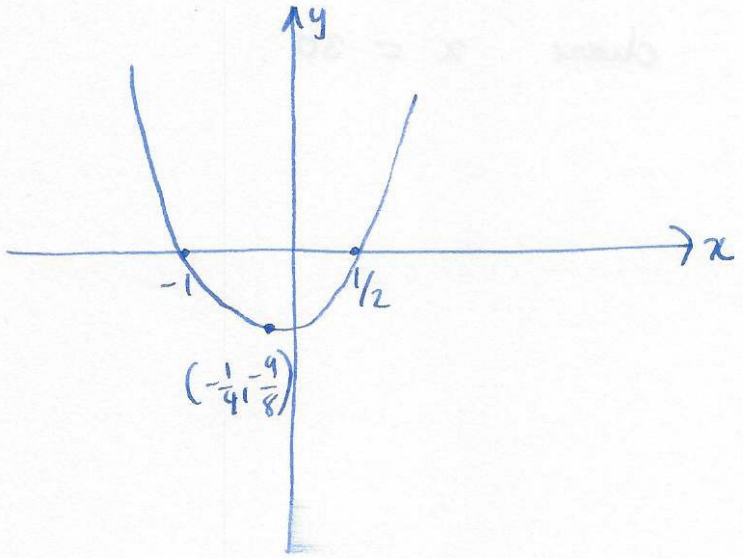
(b) (2 points) Find the coordinates of the ~~maximum~~ minimum point on the graph.

$\left(-\frac{1}{4}, -\frac{9}{8}\right)$

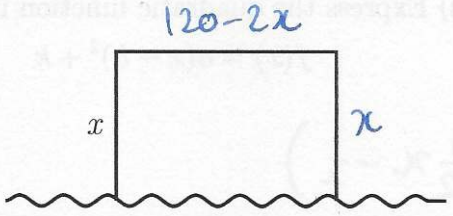
(c) (2 points) Find the x-intercepts

$(2x - 1)(x + 1)$ $\qquad x = -1, \frac{1}{2}$

(d) (2 pt) Sketch the graph of f.



(4) (**8 points**) A farmer has 120 feet of fencing, and wishes to build three fences to create a rectangular fields with one boundary by a river, as illustrated below.



(a) (2 points) Let x be the length of the fence perpendicular to the river. Write down a formula for the total area $A(x)$ of the field in terms of x.

$$A(x) = x(120 - 2x)$$
$$= -2x^2 + 120x$$

(b) (6 points) How should the farmer choose the value of x in order to maximize the total area $A(x)$?

$$A(x) = -2(x^2 - 60x)$$
$$= -2((x - 30)^2 - 900)$$
$$-2(x^2 - 60x + 900 - 900)$$

clearly $x = 30$.

(5) (**8 points**) Consider the polynomial $P(x) = x^6 + x^4 - 12x^2$.

(a) (6 points) Find all zeros (real and complex) of the polynomial $P(x)$.

$x^2(x^4 + x^2 - 12)$

$x^2(x^2 - 3)(x^2 + 4)$

$x = 0 \qquad x = \pm\sqrt{3} \qquad x = \pm 2i$

(b) (2 points) Write $P(x)$ as a product of linear polynomials.

$x^2(x - \sqrt{3}x)(x + \sqrt{3}x)(x - 2i)(x + 2i)$

(6) (**8 points**) You put \$400 in a bank account with 12% interest per year.

(a) (2 points) If the interest is compounded **monthly**, how much will you have after 1 month ?

$$A\left(\tfrac{1}{12}\right) = 400\left(1+\frac{0.12}{12}\right)^{12\cdot\frac{1}{12}} = 404$$

(b) (2 points) If the interest is compounded **continuously**, how much will you have after 12 months ?

$$A(1) = 400\, e^{0.12\cdot 1} \approx 457.00$$

(c) (4 points) If the interest is compounded **continuously**, how long will it take for you to have \$1000 ?

$$400\, e^{0.12t} = 1000$$

$$e^{0.12t} = \frac{1000}{400} = \frac{5}{2}$$

$$0.12t = \ln\left(\tfrac{5}{2}\right)$$

$$t = \frac{\ln\left(\frac{5}{2}\right)}{0.12} \approx 7.64 \text{ years}$$

(7) (**8 points**) Solve the following equations

(a) (4 points) $e^{2x} + e^x - 6 = 0$

$$(e^x - 2)(e^x + 3) = 0$$

$e^x = 2 \Rightarrow x = \ln(2)$

$e^x = -3$ no solution.

(b) (4 points) $\log_4(x+6) - \log_4(x-1) = 2$

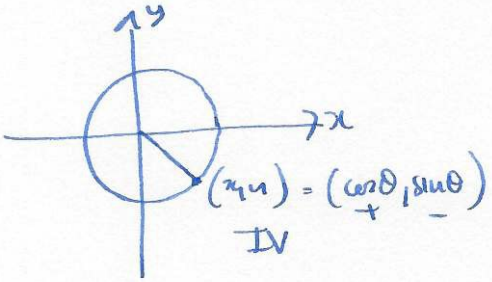
$$\frac{x+6}{x-1} = 4^2 = 16$$

$$x + 6 = 16x - 16$$

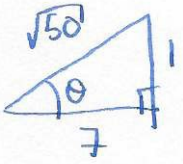
$$22 = 15x$$

$$x = \frac{22}{15}$$

(8) (**8 points**) Let θ be an angle with $\cot\theta = -7$, and with the terminal point of θ in the fourth quadrant. Calculate exact values of all six trigonometric functions of θ.



$\cot\theta = -7$ $\quad \tan\theta = -\frac{1}{7}$



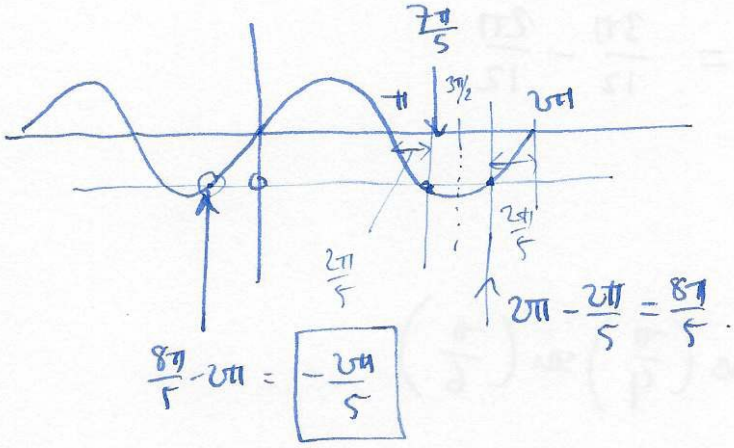
$\sin\theta = -\frac{1}{\sqrt{50}}$ $\quad \operatorname{cosec}\theta = -\sqrt{50}$

$\cos\theta = \frac{7}{\sqrt{50}}$ $\quad \sec\theta = \frac{\sqrt{50}}{7}$

(9) (**8 points**) Find the exact values of

(a) $\sin^{-1}(\sin(17\pi/5))$

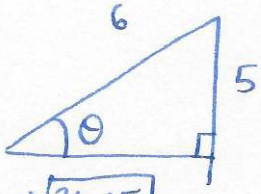
$\frac{17\pi}{5} = 3\pi + \frac{2\pi}{5}$ so $\sin\left(\frac{17\pi}{5}\right) = \sin\frac{7\pi}{5}$.



$2\pi - \frac{2\pi}{5} = \frac{8\pi}{5}$.

$\frac{8\pi}{5} - 2\pi = \boxed{-\frac{2\pi}{5}}$

(b) $\tan(\sin^{-1}(5/6))$



$\sqrt{36-25}$

$= \sqrt{11}$

$\tan\theta = \frac{5}{\sqrt{11}}$.

(10) (**8 points**) Find the exact value of $\sin(\pi/12)$. You may use the facts that

$$\sin(\pi/4) = \frac{\sqrt{2}}{2} \quad \text{and} \quad \sin(\pi/6) = \frac{1}{2}$$

$\frac{\pi}{4} = \frac{3\pi}{12}$ $\frac{\pi}{6} = \frac{2\pi}{12}$ $\frac{\pi}{12} = \frac{3\pi}{12} - \frac{2\pi}{12}$

$$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)$$

$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

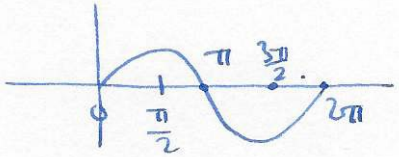
$$= \frac{\sqrt{6} - \sqrt{2}}{4}.$$

(11) (**8 points**) Find **all** solutions to the equation

$$2(\sin x)^2 + \sin x - 1 = 0.$$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$



$x = \frac{3\pi}{2} + 2\pi n$.

$x = \frac{\pi}{6}, \frac{5\pi}{6} + 2\pi n$.

(12) (**8 points**) Prove the following identity

$$\csc\theta\cos^2\theta = \csc\theta - \sin\theta.$$

$$\frac{1}{\sin\theta}\cos^2\theta = \frac{1-\sin^2\theta}{\sin\theta} = \csc\theta - \sin\theta .$$

END OF EXAMINATION