Linear Algebra Spring 13 Sample Midterm 2

(1) Suppose A is an $m \times n$ matrix for which $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has no solutions and $Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has exactly one solution.

(a) Give all possible information about m and n and the rank r of A.
(b) Find all solutions to Ax = 0 and explain your answer.
(c) Write down an example of a matrix A that fits the description in part (a).

(2) This 3×4 matrix depends on c:

$$A = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 3 & c & 2 & 8 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

(a) For each c find a basis for the column space of A.
(b) For each c find a basis for the nullspace of A.
(c) For each c find the complete solution x to $Ax = \begin{bmatrix} 1 \\ c \\ 0 \end{bmatrix}$.

(3) If A is a 3 by 5 matrix, what information do you have about the nullspace of A?

(4) Suppose row operations on A lead to this matrix $R = \texttt{rref}(A)$:

$$R = \begin{bmatrix} 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Write all known information about the columns of A.

(5) In the vector space M of all 3 by 3 matrices (you could call this a matrix space), what subspace S is spanned by all possible row reduced echelon forms R?

(6) (a) What can you say about the number of vectors in a spanning set for $\mathbb{R}^n$ which is not a basis?

(b) What can you say about the number of vectors in a linearly independent set in $\mathbb{R}^n$ which is not a basis?

(7) Let S be the following set of vectors.

$$S = \{ \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} \}$$

(a) Find a subset of S which is a basis for the span of S.
(b) Do the vectors in S span $\mathbb{R}^3$?

(8) Let v be a vector in $\mathbb{R}^n$. Show that the collection of all vectors perpendicular to v forms a vector space.

(9) $A = \begin{bmatrix} 1 & 1 & 0 & -2 & 3 \\ 0 & -1 & 1 & 3 & 1 \\ 2 & -1 & 3 & 5 & 9 \end{bmatrix}$

(a) Find the row echelon form for A.
(b) Find the rank and nullity of A.
(c) Find a basis for the image of A.
(d) Find a basis for the kernel of A.

(10) (a) Write down a 2×2 matrix giving a rotation about angle $\pi/6$ anti-clockwise.
(b) Write down a 2×2 matrix giving reflection in the x-axis.
(c) Use your previous answers to Write down a 2×2 matrix giving reflection in the line through the origin making an angle of $\pi/6$ with the x-axis.