Linear Algebra Spring 13 Sample midterm 1

(1) Explain why the following statements are true.
 (a) If A and B are inertible $n \times n$ matrices, then AB is also invertible.
 (b) An invertable matrix has a unique inverse.
 (c) If A and B are symmetric matrices, then AB is also symmetric.

(2) Write "impossible" or give an example of:
 (a) A 3×3 matrix with no zeros but which is not invertible.
 (b) A system with two equations and three unknowns that is inconsistent.
 (c) A system with two equations and three unknowns that has a unique solution.
 (d) A system with two equations and three unknowns that has infinitely many solutions.

(3) (a) Describe the intersection of the three planes in $\mathbb{R}^4$, $u+v+w+z=6$, $u+w+z=4$ and $u+w=2$.
 (b) Now describe the intersection of the three planes with $u=-1$.
 (c) Find a fourth plane such that there all four planes have empty itentersection.

(4) Consider the following linear system:

$$\begin{cases} -x_1+2x_2-x_3 & =1 \\ -x_2+2x_3 & =0 \\ 2x_1-x_2+x_4 & =0 \end{cases}$$

Write its associated augmented matrix. Reduce the matrix to its row-echelon form. Use the procedure to solve the system.

(5) Let

$$A=\begin{bmatrix} 2 & 1 & 3 \\ -4 & 2 & 1 \\ 0 & -1 & 5 \end{bmatrix} \quad B=\begin{bmatrix} 3 & 0 & 0 \\ 1 & -1 & 2 \\ 5 & 1 & 0 \end{bmatrix}$$

Compute $A+B$, AB, B^T , $\det(A)$. Recall that the determinant is the product of the pivots in the row-echelon form.

(6) Let A be the matrix

$$A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Find the LU factorization of A by writing down the elementary matrices for the row operations, and multiplying them together.

(7) Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

(a) Write down a matrix that swaps the first and second rows of A, and adds twice the first row to the third row. Show this works by writing out the matrix multiplication.
(b) Write down a matrix that swaps the first and last columns of A and adds three times the second row to the third row. Show this works by writing out the matrix multiplication.

(8) Use row operations to find the inverse of:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & -2 \end{bmatrix}$$

(9) Find examples of 2×2 matrices such that
(a) $A^2 = -I$
(b) $A^2 = 0$, with $A \neq 0$
(c) $CD = -DC \neq 0$
(d) $EF = 0$, though no entries of E or F are zero.

(10) Are the following vectors linearly independent?
(a) { [1, 2, 3], [0,0,0], [-1, 2, 0] }
(b) { [1, 2, 3], [-1, 0, 1], [2, 4, 1] }