Sample midterm 1 Solutions

Q1 a) $(AB)^{-1} = B^{-1}A^{-1}$

b) suppose A has inverses B and C. Then $AB = I = AC$

so $\underbrace{BA}_{I}B = \underbrace{BA}_{I}C \Rightarrow B = C.$

c) $(AB)^T = B^TA^T = BA \neq \cancel{A}AB$ in general

(False).

Q2 a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b) $\begin{aligned} x+y+z &= 0 \\ x+y+z &= 1 \end{aligned}$ c) impossible d) $\begin{aligned} x+y+z &= 0 \\ x+y+z &= 0. \end{aligned}$

Q3 a) $\left.\begin{aligned} u+v+w+z &= 6 \\ u+w+z &= 4 \\ u+w &= 2 \end{aligned}\right\}\ \left.\begin{aligned} v &= 2 \\ z &= 2 \end{aligned}\right\}$ so gives line with $v = 2 = z$ and $u+w = 2$

parametric form: $\begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

b) $u = -1 \Rightarrow w = 3$ single point $\begin{bmatrix} u \\ v \\ w \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$.

c) $u + w = 0$.

Q4
$$\left[\begin{array}{cccc|c} -1 & 2 & -1 & 0 & 1 \\ 0 & -1 & 2 & 0 & 0 \\ 2 & -1 & 0 & 1 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{cccc|c} -1 & 2 & -1 & 0 & 1 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 3 & -2 & 1 & 2 \end{array}\right] \rightsquigarrow \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} -1 & 2 & -1 & 0 & 1 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 4 & 1 & 2 \end{array}\right] \\ \begin{array}{cccc} & & & t \end{array} \end{array}$$

$x_4 = t$, $4x_3 + t = 2 \Rightarrow x_3 = \frac{2-t}{4}$, $-x_2 + 2\left(\frac{2-t}{4}\right) = 0 \Rightarrow x_2 = 1 - \frac{t}{2}$,

$-x_1 + 2 - t - \frac{1}{2} + \frac{t}{4} = 1. \Rightarrow x_1 = \frac{1}{2} - \frac{3}{4}t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ 1/2 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3/4 \\ -1/2 \\ -1/4 \\ 1 \end{bmatrix}$$

Q5 $A+B = \begin{bmatrix} 2 & 1 & 3 \\ -4 & 2 & 1 \\ 0 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 1 & -1 & 2 \\ 5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 & 3 \\ -3 & 1 & 3 \\ 5 & 0 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 22 & 2 & 2 \\ -5 & -1 & 4 \\ 0 & 6 & -2 \end{bmatrix} \qquad B^T = \begin{bmatrix} 3 & 1 & 5 \\ 0 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix}$$

Row reduce A:

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 7 \\ 0 & -1 & 5 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 5-7/4 \end{bmatrix} \qquad \det(A) = 2 \cdot 4 \cdot \frac{13}{4} = 26.$$

$$= 13/4.$$

Q6

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}_{L_1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{L_2.} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

so

$$L_2 L_1 A = U$$

$$A = LU \text{ where } L = L_1^{-1} L_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q7

a)

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 3 & -1 & 3 & -1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 2 & 7 & 0 \\ -1 & -1 & -2 & 1 \end{bmatrix}$$

Q8

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 3 & 1 & -2 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 & -2 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 1 & -2 \\ -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 2 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

check $$\begin{bmatrix} -1 & 0 & 0 \\ -1 & 2 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -2 & 0 & 1 \\ 3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q9 a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & +1 \\ -1 & -1 \end{bmatrix}$

Q10 a) no
contains zero

b) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & 4 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & -5 \end{bmatrix}$ yes.