

Math 233 Calculus 3 Spring 13 Sample midterm 1

- Let $\vec{u} = \langle 3, -2, 4 \rangle$ and $\vec{v} = \langle -3, 2, 1 \rangle$.
 - Find $\|\text{proj}_{\vec{v}}\vec{u}\|$.
 - Express \vec{u} as the sum of $\vec{m} = \vec{u}_{\parallel}$ parallel to \vec{v} , and $\vec{n} = \vec{u}_{\perp}$ orthogonal to \vec{v} .
- Consider three points $A(1, -1, -2)$, $B(3, -2, 2)$, $C(4, 2, 5)$.
 - Find the area of the triangle formed by A, B, C .
 - Find the equation of the plane that contains A, B, C .
- Find the equation of the plane that passes through the point $(3, -1, -2)$ and is perpendicular to the line $\langle -2, 1, -2 \rangle + t\langle 3, 1, -2 \rangle$.
- For each equation below, sketch the surface in \mathbb{R}^3 that it describes.
 - $4x^2 + y^2 + 4z^2 = 16$
 - $z = 4x^2 - 9y^2$
 - $9x^2 + 4z^2 = 2y^2 - 72$
 - $4x^2 - 9y^2 = 72$
- A particle starts at location $2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ with initial velocity $4\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 3\mathbf{k}$. Find the location of the particle at $t = 2$.
- A string in the shape of a helix has a height of 8 cm and makes four full rotations over a circle of radius 4 cm.
 - Find a parametrization $\mathbf{r}(t)$ for the string.
 - Compute the length of the string.
- Show that the following limit does not exist.
$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy - y^2}{x^2 + y^2}$$
- Find all the second order partial derivatives of
$$f(x, y, z) = ye^{x+yz} + (x + z) \cos(y + z)$$

You may assume that mixed partials are equal.
- Find the equation of the tangent plane to the surface $z = 4x^2 - 2y^2$ at the point $(2, 1, -1)$.
- Find the linear approximation to the function $f(x, y, z) = e^{4xz} + \ln(y + z)$ at the point $(1, 2, 4)$.
- Find the normal vector to the surface $z^2 = x^2 - 3y^2$ at the point $(1, -1, -2)$.