Math 214 Statistics Fall 13 Midterm 2b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

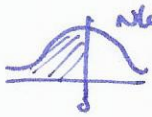
1. (10 points) Assume that the number of hours students spend on the internet per day is normally distributed with mean $\mu = 3$ and standard deviation $\sigma = 2$.

   (a) What is the probability that a randomly selected student spends less than 2 hours on the internet per day?

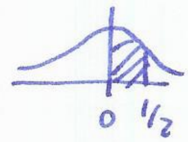
   (b) What proportion of students spend between 3 and 4 hours on the internet per day?

a) $z = \frac{x-\mu}{\sigma} = \frac{2-3}{2} = -\frac{1}{2}$ probability $= 1 - 0.6915 = 0.3085$

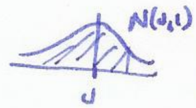
b) $\frac{3-3}{2} = 0 \leftrightarrow$



prob $1/2$



$\frac{4-3}{2} = \frac{1}{2} \leftrightarrow$



prob 0.6915

probability $= 0.6915 - 0.5$
$= 0.1915$

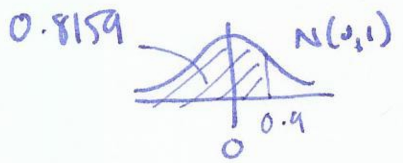
2. (10 points) Assume that the number of hours students spend on the internet per day is normally distributed with mean $\mu = 3$ and standard deviation $\sigma = 2$. Suppose a sample of 36 students is selected.

   (a) What is the distribution of the sample mean $\overline{x}$?

   (b) What is the probability that the sample mean $\overline{x}$ is greater than 3.3?

a) $\bar{x}$ dist as $N\left(3, \frac{2}{\sqrt{36}}\right) = N\left(3, \frac{1}{3}\right)$

b) $z = \frac{\bar{x}-\mu}{\sigma} = \frac{3.3-3}{1/3} = 0.9$



probability $= 1 - 0.8159 = 0.1841$

3. (10 points) Suppose 400 randomly sampled students are asked to rate the quality of donuts on campus on a scale of 1 to 10. The sample mean is 7.4. Assume that the population standard deviation is $\sigma = 4$.

   (a) Find a 90% confidence interval for the population mean.

   (b) How big a sample would be needed to make your confidence interval twice as accurate?

a) $\bar{x} \pm z_{*}\, \sigma/\sqrt{n}$ $\quad$ $7.4 \pm 1.65 \times 4/20$

$7.4 \pm \cancel{0.5445}$

0.33

$\cancel{(6.8555, 7.9445)}$ $\quad$ $(7.07, 7.73)$

b) $4 \times 400 = 1600$

4. (10 points) Suppose 400 randomly sampled students are asked to rate the quality of donuts on campus on a scale of 1 to 10. The sample mean is 7.4. Assume that the population standard deviation is $\sigma = 4$. The donut shop claims that their donuts are rated as 8 out of 10 by the student body. Use the sample to test this claim at the 10% significance level.

(a) State the null hypothesis and the alternate hypothesis.

(b) Specify the test statistic and its distribution.

(c) Calculate the test statistic.

(d) Would you reject or fail to reject H_0 at the given significance level? State your conclusion.

a) $H_0 : \mu = 8$

$H_a : \mu \neq 8$

b) $\dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$, critical value $= -1.65$

c) $\dfrac{7.4 - 8}{4 / \sqrt{400}} = \dfrac{-0.6}{1/5} = -3$

d) critical value -1.65, $\quad -3 < -1.65$ reject H_0

significant evidence rating is not 8.

5. (10 points) A machine in the tortilla chip factory is meant to fill 16oz bags. For quality control purposes a random sample of 25 bags is selected and weighed. The mean weight of each bag in the sample is $\overline{x} = 15.6$oz, and the standard deviation of the sample is $s = 1.2$oz.

   (a) Find a 99% confidence interval for the average weight of a bag filled by the machine.

   (b) Is the machine working correctly at this confidence level?

a) $\bar{x} \pm t_* \, s/\sqrt{n}$ $\qquad$ t_* critical value for t-dist with 24 degrees of freedom

$15.6 \pm 2.797 \times 1.2/5 = 15.6 \pm 0.6713$

$(14.9287, 16.2713)$

b) Yes, 16 lies in the confidence interval.

6. (10 points) In a sample of 16 football players the average calorie consumption was 3077 kcal/day, with a standard deviation of 987 kcal/day. The recommended amount is 3422. Is there evidence that the football players are not eating enough?

(a) State the null hypothesis and the alternate hypothesis.

(b) Specify the test statistic and its distribution.

(c) Calculate the test statistic.

(d) Would you reject or fail to reject H_0 at the 10% significance level? State your conclusion.

a) $H_0: \mu = 3422$

$H_a: \mu < 3422$

b) $\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim$ ~~t-dist~~ t-dist with 15 degrees of freedom

c) $\frac{3077 - 3422}{987/4} = -1.3982$

d) critical value -1.341, reject H_0

significant evidence player not eating enough

7. (10 points) Cholesterol levels were measured for random sample of male and female patients.

	sample size n	$\overline{x}$	s
Female	40	174	30
Male	30	171	33

Test whether this is evidence that the female and male patents have different cholesterol levels, at the 5% signficicance level.

(a) State the null hypothesis and the alternate hypothesis.

(b) Specify the test statistic and its distribution.

(c) Calculate the test statistic.

(d) Would you reject or fail to reject H_0 at the given significance level? State your conclusion.

a) $H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$

b) $\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$ dist. as t-dist with 29 degrees of freedom

c) $\frac{174-171}{\sqrt{30^2/40 + 33^2/30}} = \frac{3}{7.681} \approx 0.3912$

d) critical value 1.699 fail to reject H_0

no significant evidence that populations have different means

8. (10 points) A simple random sample of 450 residents in the state of New York is taken to estimate the proportion of people who live within one mile of a hazardous waste site, and 180 of the residents in the sample live within one mile of a hazardous waste site.

   (a) What is the value of the sample proportion of people who live within one mile of a hazardous waste site?

   (b) What is the standard error for the proportion of people who live within one mile of the hazardous waste site?

   (c) Find a 95% confidence interval for the proportion of people who live within one mile of the hazardous waste site.

a) $\hat{p} = \frac{180}{450} = 0.4$

b) $s = \sqrt{0.4(1-0.4)/450} = \cancel{0.34} \quad 0.02309$

c) $\hat{p} \pm z_* s = 0.4 \pm 1.96 \times 0.02309$

$= (0.359547, 0.4453)$

9. (10 points) The candy company that makes M&Ms claims that 10% of the M&Ms it produces are green. Suppose that the candies are packaged at random in large bags of 200 M&Ms. When we randomly pick a bag of M&Ms we may assume that this represents a simple random sample of size $n = 200$. The bag we chose contains 22 green M&M's. We wish to test the company's claim at the 5% significance level.

   (a) State the null hypothesis and the alternate hypothesis.

   (b) Specify the test statistic and its distribution.

   (c) Calculate the test statistic.

   (d) Would you reject or fail to reject H_0 at the given significance level? State your conclusion.

a) $H_0: p = 0.1$
$H_a: p \neq 0.1$

b) $z = \dfrac{\hat{p} - p}{\sqrt{p(1-p)/n}}$ dist as $N(0,1)$

c) $\dfrac{\frac{22}{200} - 0.1}{\sqrt{0.1 \times 0.9 / 200}} = \dfrac{0.01}{0.02121} = 0.4714$

d) critical value 1.96 ~~reject H_0: significant evidence proportion is not 0.1~~

fail to reject H_0: no significant evidence proportion is not 0.1.

10. (10 points) A survey of pet owners in New York and New Jersey obtained the following results

	sample size n	number of pet owners
New Jersey	65	40
New York	85	40

Is this evidence that the two states have different proportions of pet owners? Test at the 10% signficance level.

(a) State the null hypothesis and the alternate hypothesis.

(b) Specify the test statistic and its distribution.

(c) Calculate the test statistic.

(d) Would you reject or fail to reject H_0 at the given significance level? State your conclusion.

a) $H_0: p_1 = p_2$

$H_a: p_1 \neq p_2$

b) $\dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$, $\hat{p} = \dfrac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2}$ dist as $N(0,1)$

c) $\dfrac{\frac{40}{65} - \frac{40}{85}}{\sqrt{0.5333 \times 0.4667 \times \left(\frac{1}{65} + \frac{1}{85}\right)}}$ $p = \dfrac{40+40}{65+85} = 0.5333$

$= \dfrac{0.1448}{0.08218} = 1.762$

d) critical value 1.65 reject H_0: significant evidence proportions are different

Formulas

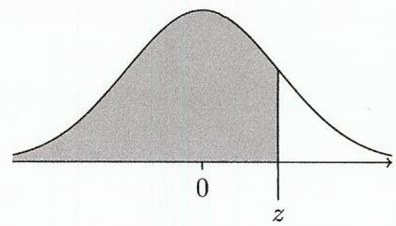
μ population mean
σ population standard deviation
n sample size
$\overline{x}$ sample mean
s sample standard deviation (standard error)
p population proportion
$\widehat{p}$ sample proportion

The sample mean $\overline{x}$ of a normal distribution $N(\mu, \sigma)$ has distribution $N(\mu, \sigma/\sqrt{n})$.

The sample mean of any distribution with mean μ and standard deviation σ has distribution approximately $N(\mu, \sigma/\sqrt{n})$, for n sufficiently large.

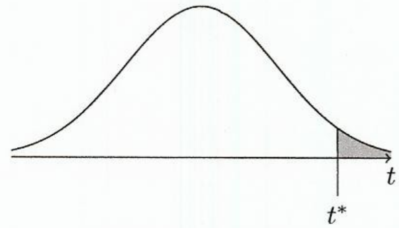
	Confidence interval	Test statistic	Distribution
mean, known σ	$\overline{x} \pm z_* \sigma/\sqrt{n}$	$\dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$	$N(0,1)$
mean, unknown σ	$\overline{x} \pm t_* s/\sqrt{n}$	$\dfrac{\overline{x} - \mu}{s/\sqrt{n}}$	t-dist, df $= n - 1$
difference between two means	$\overline{x_1} - \overline{x_2} \pm t_* \sqrt{s_1^2/n_1 + s_2^2/n_2}$	$\dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$	t-dist, df=$\min\{n_1, n_2\} - 1$
proportion	$\widehat{p} \pm z_* \sqrt{\widehat{p}(1-\widehat{p})/n}$	$\dfrac{\widehat{p} - p}{\sqrt{p(1-p)/n}}$	$N(0,1)$
difference between two proportions	$\widehat{p}_1 - \widehat{p}_2 \pm z_* \sqrt{\widehat{p}(1-\widehat{p})(1/n_1 + 1/n_2)}$, where $\widehat{p} = (\widehat{p}_1 n_1 + \widehat{p}_2 n_2)/(n_1 + n_2)$	$\dfrac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p})(1/n_1 + 1/n_2)}}$	$N(0,1)$

Standard normal distribution table



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

t-distribution critical values



	Upper tail probability p (one-sided)											
df	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0200	0.0100	0.0050	0.0025	0.0010	0.0005
	Upper tail probability p (two-sided)											
df	0.5000	0.4000	0.3000	0.2000	0.1000	0.0500	0.0400	0.0200	0.0100	0.0050	0.0020	0.0010
1	1.000	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	127.321	318.309	636.619
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.089	22.327	31.599
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.215	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291