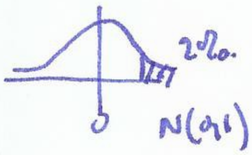
Sample midterm 2 Solutions

Q1 $X \sim N(20, 0.15)$ $\qquad X = 0.15Z + 20 \iff Z = \dfrac{X-20}{0.15}$

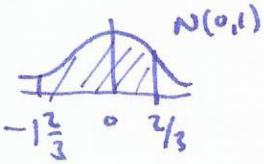
a) $Z \sim N(0,1)$



$\rightsquigarrow Z = 2.0537 \Rightarrow X = 20.3081$

exclude gas tanks ≥ 20.3081

b) $19.75 \leq X \leq 20.10 \iff \dfrac{19.75-20}{0.15} \leq Z \leq \dfrac{20.10-20}{0.15}$



$-1.6667 \leq Z \leq 0.6667$

$\Rightarrow$ proportion is 69.97%

c) sample mean $\bar{X} \sim N\left(20, \dfrac{0.15}{\sqrt{16}}\right)$ $\qquad Z = \dfrac{X-20}{0.15/4}$

$$19.75 \leq \bar{X} \leq 20.10$$

$$\frac{19.75-20}{0.15/4} \leq Z \leq \frac{20.10-20}{0.15/4}$$

$-6.6667 \leq Z \leq 2.6667 \Rightarrow$ probability is 99.62%

Q2 a) $500 \times 0.9 = 450 = \mu = np$

b) $\sigma = \sqrt{np(1-p)} = \sqrt{\cancel{500}\,100 \times 0.9 \times 0.1} = \cancel{6.7082}\ 3$.

c) $\binom{100}{99} \cancel{0.9^{9}} (0.9)^{99} (0.1) = 100\,(0.9)^{99}(0.1) = 0.0002951$

d) $B(500, 0.9)$ approx $N\left(np, \sqrt{np(1-p)}\right) = N(450, 6.7082)$

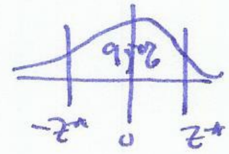
$X = 440 \iff Z = \dfrac{440-450}{6.7082} = -1.4907$

prob = 93.2%

Q3

a) $X \sim N(\mu, 10)$ $\quad \bar{x} \sim N(\mu, \frac{10}{\sqrt{n}})$ $\quad n = 36$

95% confidence interval is $\bar{x} \pm z^* \frac{10}{6}$



$z^* = 1.96$ $\quad 104 \pm 3.2667 = (100.7333, 107.2667)$

b) $36 \times 4 = 144$

Q4 a) H_0: average rent $\mu = \$650$

H_a: average rent $\mu > \$650$. (or $\mu \neq \$650$).

b) Fail to reject H_0: ~~it is 95% likely that that~~ only reject H_0 if the probability the sample came from a dist with $\mu = \$650$ is less than 5%.

(note: should really use 1-sided test here!).

Q5 a) H_0: mean stopping distance $\mu = 215$

H_a: mean stopping distance $\mu < 215$

b) test statistic: $\frac{\bar{x} - 215}{2.15/3} \sim N(0,1)$

c) $z = -2.7907$

d) p-value: $0.002630 = 0.26\%$

e) reject H_0 at 5% confidence level: new tires are significantly better at 5% significance level.

Q6 a) H_0: no difference in population ~~proportion~~ ~~means~~ means; $\cancel{p_1 = p_2 = p}$ $\mu_1 = \mu_2$

H_a: $\cancel{p_1 \neq p_2}$, population ~~proportions~~ means different.

$\mu_1 \neq \mu_2$

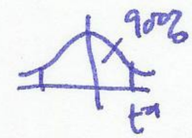
b) $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ dist as t-dist with $\min\{n_1-1, n_2-1\}$ degrees of freedom

c) $\dfrac{11.1 - 5.8}{\sqrt{\dfrac{(5.04)^2}{20} + \dfrac{5^2}{15}}} \simeq 3.0927$

d) p-value from R: pt(3.0927, df = 14) = 0.9960

e) reject ~~a~~ H_0 at 5% significance level, significant evidence pills have different means.

<u>Q7</u> a) confidence interval is $\bar{x} \pm t^* \, s/\sqrt{n}$ where t^* is



from a t-dist with $(n-1)$-degrees of freedom.

$126 \pm (1.7109) \times 4.2/5 = (124.56, 127.44)$

b) No. 120 does not lie in the 90% confidence interval

<u>Q8</u> a) $H_0: \mu = 48$, new standard is accurate

$H_a: \mu \neq 48$, new standard is not accurate

b) test statistic = $\dfrac{\bar{x} - \cancel{48}\,\mu}{s/\sqrt{n}}$ dist as a t-dist with $(n-1)$-degrees of freedom

$\dfrac{51.5 - 48}{(3.37/\sqrt{10})} \sim$ t-dist with 9-degrees of freedom

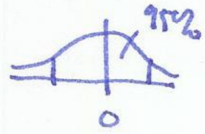
c) $t = \cancel{4}\ 3.2843$

d) P-value (from R: pt(3.2843, df = 9)) is 0.99527

e) Reject H_0 at 5% significance level. Significant evidence that new standards are inaccurate.

Q9 a) $\hat{p} = 51/85 = 0.6$ $\quad SE_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})/n} = 0.05314$

95%-confidence interval $\quad \hat{p} \pm z_* SE_{\hat{p}}$ $\quad z_*$ from $N(0,1)$



$z_* = 1.96$

$0.6 \pm 1.96 \times 0.05314 = (0.4958, 0.7042)$

b) $H_0 : \hat{p} = 0.5$ half of all students have parents who buy them cars

$H_a : \hat{p} \neq 0.5$ proportion is not 0.5.

(or $\hat{p} > 0.5$)

c) test statistic $\quad z = \dfrac{\hat{p} - 0.5}{\sqrt{0.5 \times 0.5 / n{=}85}} = \dfrac{0.6 - 0.5}{\sqrt{(1/4) \times \frac{1}{85}}} = 1.8439$

(dist as $N(0,1)$)

d) p-value: 0.03260

e) reject H_0 at 5% confidence level: significant evidence that proportion is not 1/2.

Q10 a) $\hat{p}_1 - \hat{p}_2 = \dfrac{20}{40} - \dfrac{21}{45} = 0.03333$

b) $H_0 : \hat{p}_1 = p_2$ two proportions are the same

$H_a : p_1 \neq p_2$ two proportions different

c) statistic is $\dfrac{\hat{p}_1 - \hat{p}_2}{SE_{pooled}}$ $\quad SE_{pooled}$: $\hat{p} = \dfrac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2} = \dfrac{20+21}{40+45} = 0.4824$

dist as $N(0,1)$ $\quad SE_{pooled} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = 0.1086$

$= \dfrac{0.03333}{0.1086} = 0.3069$

d) p-value = 0.3794

e) fail to reject H_0: no significant evidence proportions are not the same.