**Math 231 Calculus 1 Fall 13 Midterm 3a**

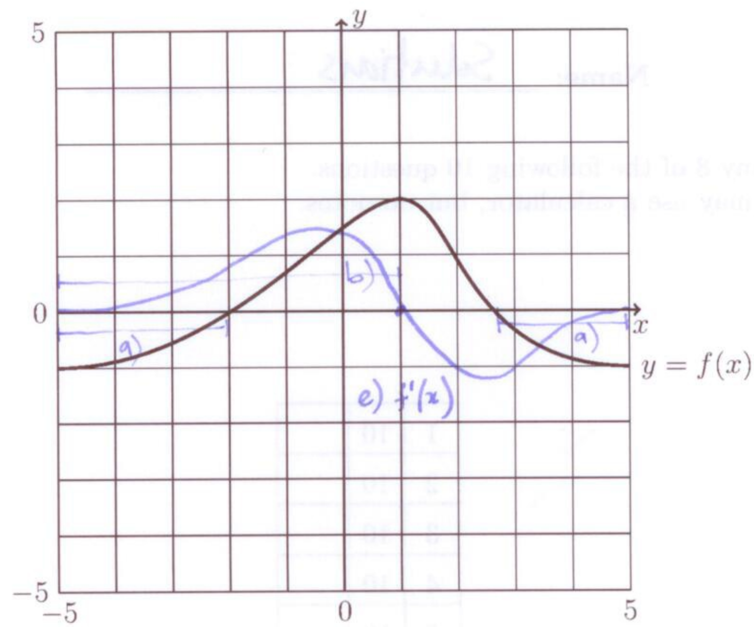
**Name:** Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 3 | |
| Overall | |

(1) (10 points) Consider the function $f(x)$ defined by the following graph.




(a) Label all regions where $f(x) < 0$.

(b) Label all regions where $f'(x) > 0$.

(c) What is $\lim_{x\to\infty} f(x)$? −1

(d) What is $\lim_{x\to\infty} f'(x)$? 0

(e) Sketch a graph of $f'(x)$ on the figure.

(2) (10 points) Consider the function $f(x) = \frac{1}{x^2 - 9}$.

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.

a) vertical asymptotes: $x^2 - 9 = 0 \Leftrightarrow x = \pm 3$

horizontal asymptotes $\lim_{x \to \pm\infty} \frac{1}{x^2-9} = 0$

b) $f'(x) = -(x^2-9)^{-2} \cdot 2x = \frac{-2x}{(x^2-9)^2}$

$f'(x) = 0$ at $x = 0$

$f'(x)$ undefined at $x = \pm 3$

c) $f'(x)$: $+$ | $-3$ | $+$ | $0$ | $-$ | $3$ | $-$

increasing, $f'(x) > 0$ $(-\infty, -3)$ and $(-3, 0)$

decreasing, $f'(x) < 0$ $(0, 3)$ and $(3, \infty)$

(3) (10 points) Consider the function $f(x) = xe^{2x}$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them.

a) $f'(x) = e^{2x} + x \cdot e^{2x} \cdot 2$

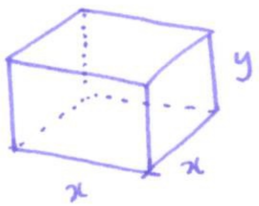
solve $f'(x) = 0$ : $e^{2x}(1+2x) = 0 \quad x = -\frac{1}{2}$

b) $f''(x) = 2e^{2x} + 2e^{2x} + x\,2e^{2x} \cdot 2$

$= e^{2x}(4+4x)$

$f''(-\frac{1}{2}) \overset{?}{>} 0 \Rightarrow$ local max

(4) (10 points) A cardboard box has a square base with sides of length $x$, and four vertical sides of height $y$, and no top. Find the dimensions of the box of volume $1\text{m}^3$ with smallest surface area.




$V = x^2 y = 1 \qquad y = \frac{1}{x^2}$

$A = x^2 + 4xy \qquad A = x^2 + \frac{4x}{x^2} = x^2 + \frac{4}{x}$

$\frac{dA}{dx} = 2x - 4x^{-2}$

solve $\frac{dA}{dx} = 0$: $\quad 2x - \frac{4}{x^2} = 0 \qquad x^3 = 2 \qquad x = \sqrt[3]{2}$

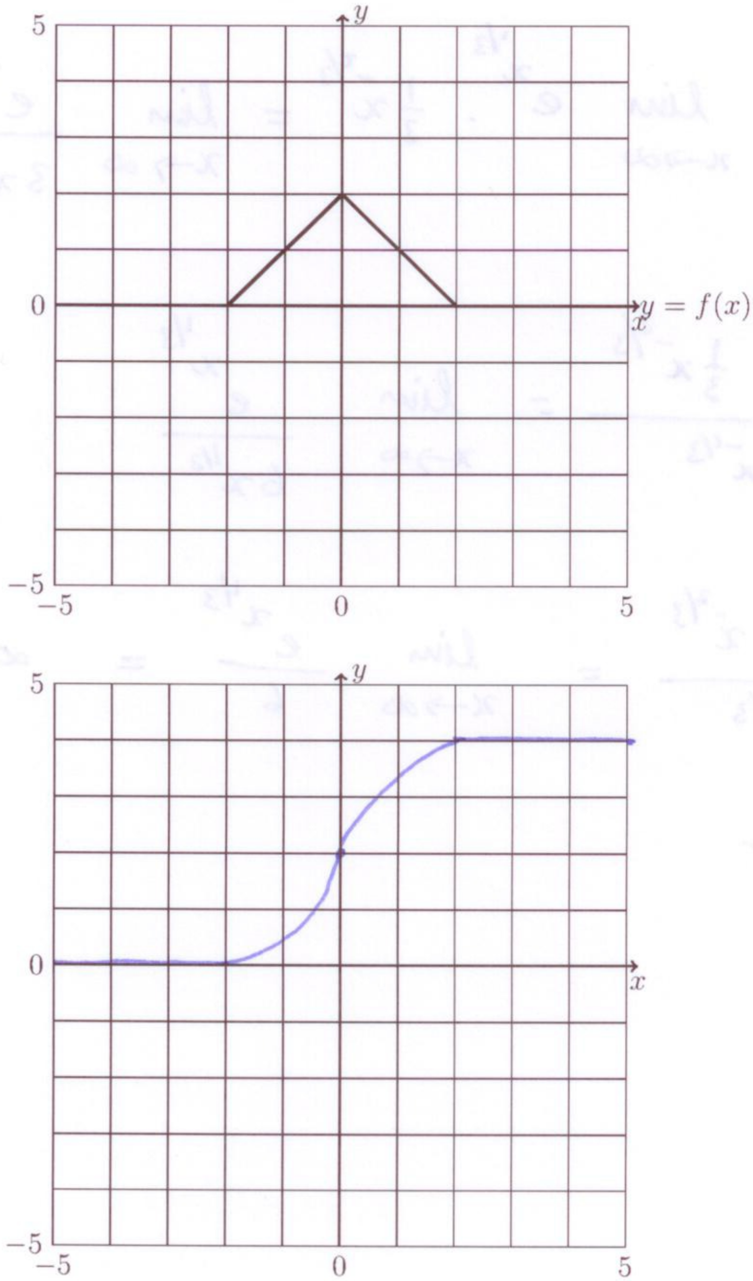
$y = \frac{1}{x^2} = \frac{1}{2^{2/3}}$

(5) (10 points) Find

$$\lim_{x\to 0} \frac{1-e^{3x}}{\sin x}.$$

l'H: $= \lim_{x\to 0} \frac{-3e^{3x}}{\cos x} = -3$

(6) (10 points) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.



(7) (10 points) Which function grows faster, $x$ or $e^{\sqrt[3]{x}}$? Justify your answer. (Hint: take a limit.)

$$\lim_{x\to\infty} \frac{e^{x^{1/3}}}{x} \underset{l'H}{=} \lim_{x\to\infty} e^{x^{1/3}} \cdot \frac{1}{3}x^{-2/3} = \lim_{x\to\infty} \frac{e^{x^{1/3}}}{3x^{2/3}}.$$

$$l'H: = \lim_{x\to\infty} \frac{e^{x^{1/3}} \cdot \frac{1}{3}x^{-2/3}}{3\frac{2}{3}x^{-1/3}} = \lim_{x\to\infty} \frac{e^{x^{1/3}}}{6x^{1/3}}$$

$$l'H: \lim_{x\to\infty} \frac{e^{x^{1/3}} \cdot \frac{1}{3}x^{-2/3}}{6\frac{1}{3}x^{-2/3}} = \lim_{x\to\infty} \frac{e^{x^{1/3}}}{6} = \infty$$

so $e^{\sqrt[3]{x}}$ grows faster

(8) (10 points) Find the indefinite integral

$$\int e^x - 4\sin(x)\ dx.$$

$e^x + 4\cos(x) + c$

(9) (10 points) Evaluate the definite integral

$$\int_1^9 \frac{\sqrt{x}+1}{x}\, dx.$$

$$\int_1^9 x^{-1/2} + x^{-1}\, dx = \left[ 2x^{1/2} + \ln|x| \right]_1^9$$

$$= 6 + \ln(9) - 2 - \ln(1)$$

$$= 4 + \ln(9)$$

(10) Find the area under the graph $y = 2x^2 - 2x$ between $x = 0$ and $x = 1$.

$$\int_0^1 2x^2 - 2x \, dx = \left[ \frac{2}{3}x^3 - x^2 \right]_0^1 = \frac{2}{3} - 1 = -\frac{1}{3}$$