Math 231 Calculus 1 Fall 13 Midterm 2b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of $f(x) = \tan^{-1}(\frac{2}{x^2})$.

$$\frac{1}{1+\left(\frac{2}{x^2}\right)^2} \cdot 2(-2)x^{-3} = \frac{-4}{x^3 + \frac{4}{x}} = \frac{-4x}{x^4+4}$$

wlg: $\frac{2}{x^2} = 2x^{-2}$ $\quad \frac{d}{dx}(2x^{-2}) = -4x^{-3}$

or $\frac{d}{dx}\left(\frac{2}{x^2}\right) = \frac{x^2 \cdot (2)' - (x^2)' \cdot 2}{x^4} = \frac{x^2 \cdot 0 - 4x}{x^4} = \frac{-4x}{x^4} = \frac{-4}{x^3}$

(2) (10 points) Find the derivative of $f(x) = e^{-2x^2}\sin(x)$.

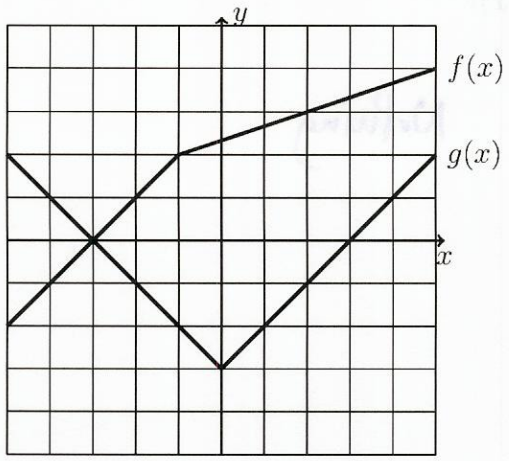
$f'(x) = e^{-2x^2} \cdot -4x \sin(x) + e^{-2x^2}\cos(x)$

(3) (10 points) Find the second derivative of $f(x) = \sqrt{3x^2 + 2}$. $= (3x^2+2)^{1/2}$

$f'(x) = \frac{1}{2}(3x^2+2)^{-1/2} \cdot 6x$

$f''(x) = -\frac{1}{4}(3x^2+2)^{-3/2} \cdot 36x^2 + 3(3x^2+2)^{-1/2}$

(4) (10 points) The graphs of the functions f and g are shown below.



(a) Let $h(x) = f(x)g(x)$ Find $h'(1)$.
(b) Let $h(x) = f(g(x))$. Find $h'(4)$.

a) $h'(1) = f'(1)g(1) + f(1)g'(1)$

$\frac{1}{3} \cdot -2 + 2\frac{2}{3} \cdot 1 = 2$

b) $h'(4) = f'(g(4)) \cdot g'(4)$

$\underbrace{f'(1)}_{1/3} \cdot 1 = \frac{1}{3}$.

(5) (10 points)

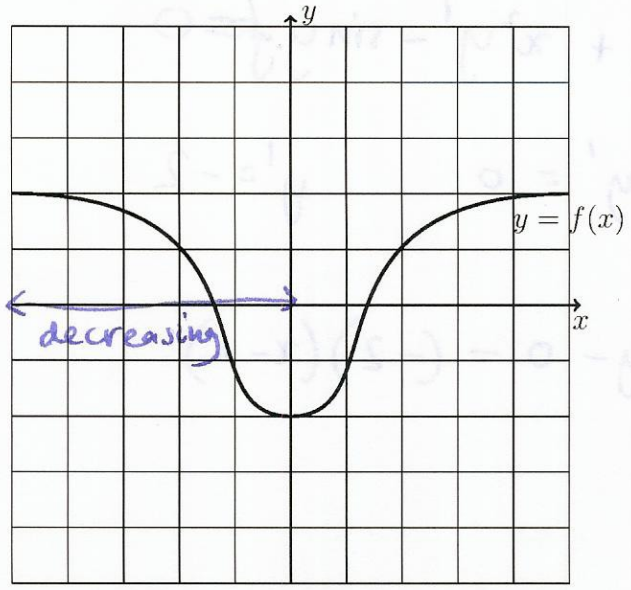
(a) Suppose a function $f(x)$ satisfies $f(x) < 0$ for all x. What can you say about $f'(x)$?

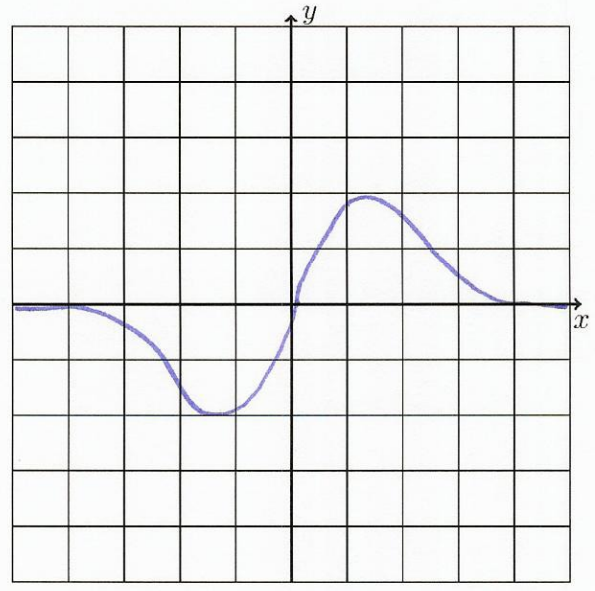
Nothing

(b) Suppose a function $g(x)$ satisfies $g'(x) > 0$ for all x. What can you say about $g(x)$?

$g'(x) > 0 \iff$ increasing

(6) (10 points) The graph of a function $f(x)$ is drawn below. On the top axes indicate where $f(x)$ is decreasing. Sketch the graph of $f'(x)$ on the lower axes.





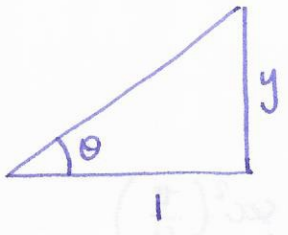
(7) (10 points) The equation $2x + x^2y + \cos y = 3$ determines a curve in the plane. Find the equation of the tangent line to the point $(1, 0)$.

$2 + 2xy + x^2y' - \sin y \cdot y' = 0$

$(1,0)$: $\quad 2 + y' = 0 \qquad y' = -2$

tangent line : $\quad y - 0 = (-2)(x - 1)$

(8) (10 points) A hot air balloon rises vertical upwards from a distance of 1 km away. When yousee the ball at an angle of $\pi/4$, the angle is changing at a rate of 0.1 radians/hour. How fast is the balloon rising?



$$\tan\theta = \frac{y}{1}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt}$$

$$\frac{dy}{dt} = \underbrace{\sec^2\left(\frac{\pi}{4}\right)}_{2} \cdot 0.1 = 0.2 \text{ km/h}$$

(9) (10 points) The value of $\tan x$ at $\pi/4$ is 1. Use linear approximation to estimate $\tan^{-1}(0.9)$. What is the percentage error in the approximation?

$f(x+h) \approx f(x) + hf'(x)$

$f(x) = \tan^{-1}(x) \qquad f'(x) = \frac{1}{1+x^2}$

$f(1) = \frac{\pi}{4} \qquad f'(1) = \frac{1}{1+1} = \frac{1}{2}$

$f(1-0.1) \approx \frac{\pi}{4} - 0.1 \cdot \frac{1}{2} \approx 0.735$

percentage error: $\dfrac{\left|\frac{\pi}{4} - 0.05 - \tan^{-1}(0.9)\right|}{\tan^{-1}(0.9)} \times 100 \approx 0.35\%.$

(10) (10 points) Find the absolute maximum and minimum of $f(x) = 2x^2 - 4x + 2$ on the interval $[-2, 2]$.

$f'(x) = 4x - 4$

solve $f'(x) = 0$: $x = 1$

check $f(-2) = 18$ max

$f(1) = 0$ min

$f(2) = 2$