Math 231 Calculus 1 Fall 13 Midterm 2a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of $f(x) = \tan^{-1}(\frac{3}{x^2})$.

$$\frac{1}{1+\left(\frac{3}{x^2}\right)^2} \cdot 3.(-2)x^{-3} = \frac{-6}{x^3+\frac{9}{x}} = \frac{-6x}{x^4+9}$$

note: $\frac{3}{x^2} = 3x^{-2}$ $\qquad \frac{d}{dx}(3x^{-2}) = -6x^{-3}$

or: $\frac{d}{dx}\left(\frac{3}{x^2}\right) = \frac{x^2(3)' - 2x.3}{x^4} = \frac{x^2.0 - 6x}{x^4} = -6x^{-3}.$

(2) (10 points) Find the derivative of $f(x) = e^{-3x^2}\sin(x)$.

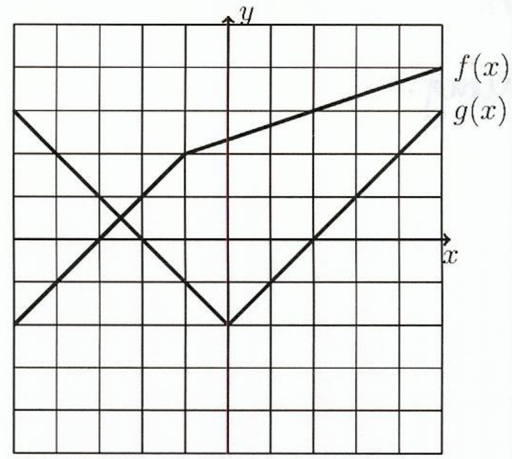
$e^{-3x^2} \cdot (-6x)\sin(x) + e^{-3x^2}\cos(x)$

(3) (10 points) Find the second derivative of $f(x) = \sqrt{2x^2+3}$. $= (2x^2+3)^{1/2}$

$f'(x) = \frac{1}{2}(2x^2+3)^{-1/2} \cdot 4x$

$f''(x) = -\frac{1}{4}(2x^2+3)^{-3/2} \cdot 16x^2 + 2(2x^2+3)^{-1/2}$

(4) (10 points) The graphs of the functions f and g are shown below.



(a) Let $h(x) = f(x)g(x)$ Find $h'(1)$.
(b) Let $h(x) = f(g(x))$. Find $h'(3)$.

a) $h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$

$h'(1) = f'(1)\,g(1) + f(1)\,g'(1)$

$\frac{1}{3} \cdot -1 + 2\frac{2}{3} \cdot 1 = \frac{7}{3}$

b) $h'(3) = f'(g(3)) \cdot g'(3)$

$= f'(1) \cdot 1 = \frac{1}{3}$

(5) (10 points)

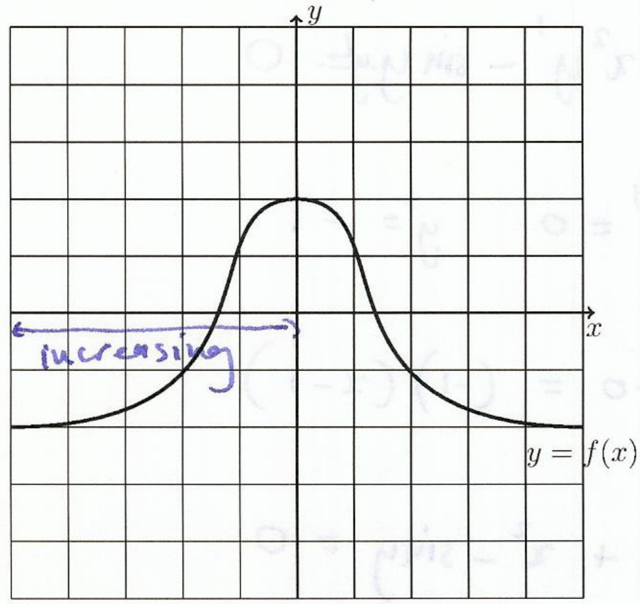
(a) Suppose a function $f(x)$ satisfies $f(x) > 0$ for all x. What can you say about $f'(x)$?

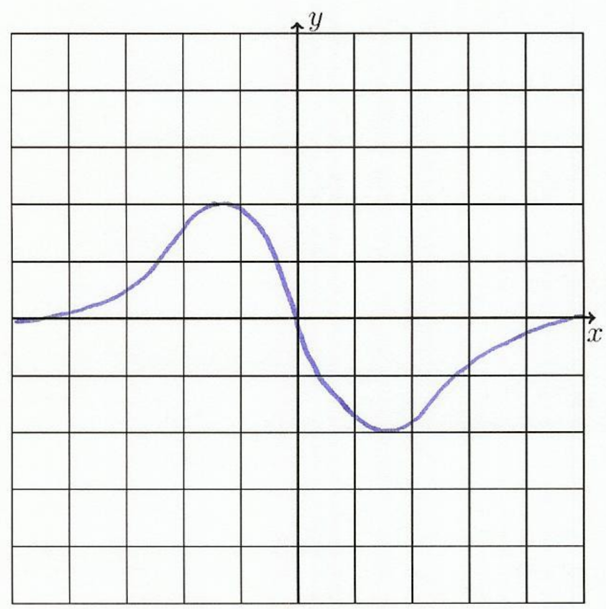
Nothing.

(b) Suppose a function $g(x)$ satisfies $g'(x) < 0$ for all x. What can you say about $g(x)$?

$g'(x) < 0 \leftrightarrow g(x)$ decreasing

(6) (10 points) The graph of a function $f(x)$ is drawn below. On the top axes indicate where $f(x)$ is increasing. Sketch the graph of $f'(x)$ on the lower axes.





(7) (10 points) The equation $x + x^2y + \cos y = 2$ determines a curve in the plane. Find the equation of the tangent line to the point $(1, 0)$.

wrt x:

$$1 + 2xy + x^2y' - \sin y \cdot y' = 0$$

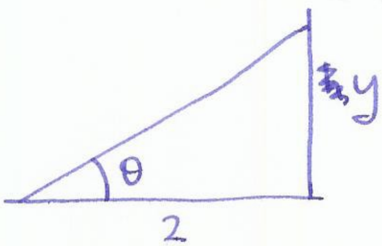
$$1 + y' = 0 \qquad y' = -1$$

$$y - 0 = (-1)(x-1)$$

wrt y: $x' + 2xx'y + x^2 - \sin y = 0$

$$\frac{dx}{dy} = -1$$

(8) (10 points) A hot air balloon rises vertical upwards from a distance of 2 km away. When yousee the ball at an angle of $\pi/6$, the angle is changing at a rate of 0.2 radians/hour. How fast is the balloon rising?



$$\tan\theta = \frac{y}{2}$$

$$\tan(\theta(t)) = \frac{y(t)}{2}$$

$$\sec^2(\theta(t))\frac{d\theta}{dt} = \frac{1}{2}\frac{dy}{dt}$$

$$\sec^2\left(\frac{\pi}{6}\right)0.2 = \frac{1}{2}\frac{dy}{dt}$$

$$\frac{4}{3}\cdot 0.4 = \frac{dy}{dt} = \frac{16}{30} \text{ km/hour}$$

(9) (10 points) The value of $\tan x$ at $\pi/4$ is 1. Use linear approximation to estimate $\tan^{-1}(0.8)$. What is the percentage error in the approximation?

$$f(x+h) \approx f(x) + hf'(x)$$

$$f(x) = \tan^{-1}(x) \qquad f'(x) = \frac{1}{1+x^2}$$

$$f(1) = \frac{\pi}{4} \qquad f'(1) = \frac{1}{1+1^2} = \frac{1}{2}$$

$$f(1-0.2) \approx \frac{\pi}{4} - 0.2 \cdot \frac{1}{2} \approx 0.6854$$

percentage error: $\dfrac{|\tan^{-1}(0.8) - 0.6854|}{\tan^{-1}(0.8)} \times 100 \approx 1.6\%$

(10) (10 points) Find the absolute maximum and minimum of $f(x) = x^2 - 4x - 2$ on the interval $[-3, 3]$.

$f'(x) = 2x - 4$

solve $f'(x) = 0$: $x = 2$

check : $f(-3) = 19$ max

$f(2) = -6$ min

$f(3) = -5$