Math 231 Calculus 1 Fall 13 Midterm 1b

Name: _____Solutions_____

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

1

(1) (10 points) Plot the points $(-4, 1)$ and $(2, -3)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.
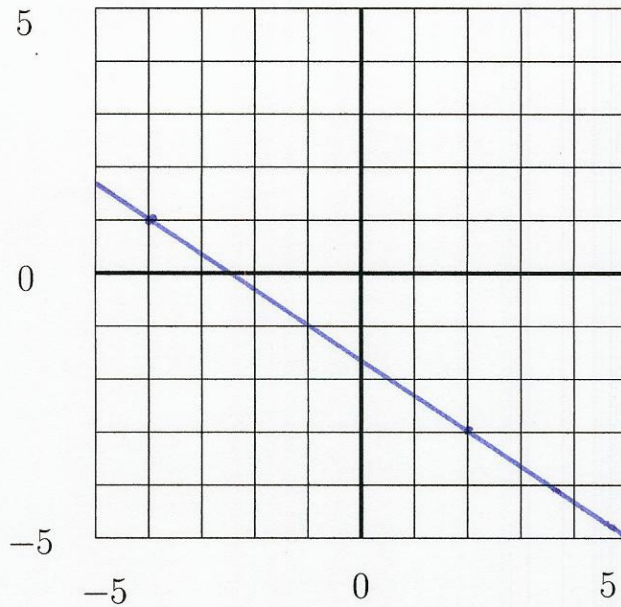

FIGURE 1

$$\text{slope} = \frac{1-(-3)}{-4-2} = \frac{4}{-6} = -\frac{2}{3}$$

$$y - 1 = -\frac{2}{3}(x+4)$$

$$y = -\frac{2}{3}x + 1 - \frac{8}{3}$$

$$y = -\frac{2}{3}x - \frac{5}{3}$$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.
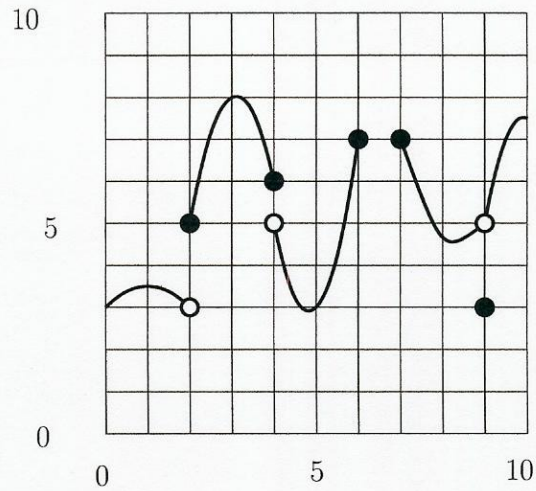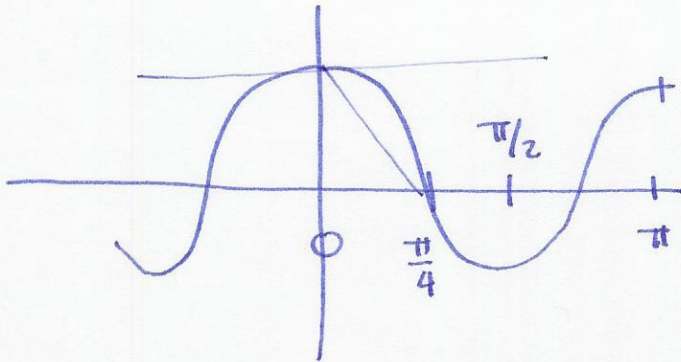

FIGURE 2. $f(x)$

(a) $\lim_{x \to 2^-} f(x)$ 3

(b) $\lim_{x \to 4} f(x)$ DNE

(c) $\lim_{x \to 6^-} f(x)$ 7

(d) $\lim_{x \to 6^+} f(x)$ DNE

(e) $\lim_{x \to 9} f(x)$ 5

4

(3) (10 points) Sketch the graph of $f(x) = \cos 2x$.
   (a) What is the average rate of change from $x = 0$ to $x = \pi/4$?
   (b) Looking at the graph, do you expect this to be bigger or smaller than the actual rate of change at $x = 0$?

a) $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{0 - 1}{\pi/4 - 0} = \dfrac{-4}{\pi}$

b) smaller

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 0} \frac{\sin 2x}{3x}$$

$2x = \theta \iff x = \theta/2$

$\lim_{\theta \to 0} \frac{\sin\theta}{3\theta/2} = \quad \frac{2}{3} \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \quad \frac{2}{3}$

(5) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$$

$$\lim_{x \to 4} \frac{(x-4)(x+3)}{(x-4)} = \lim_{x \to 4} x+3 = 7$$

(6) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 9} \frac{\cancel{36}^{\,6}}{x - 9} - \frac{1}{\sqrt{x} - 3}$$

$$\lim_{x \to 9} \frac{6\cancel{x} - (\sqrt{x} + 3)}{(\sqrt{x} + 3)(\sqrt{x} - 3)} = \lim_{x \to 9} \frac{-1}{\sqrt{x} + 3} = \frac{-1}{6}$$

(7) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 4x$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) - x^2 + 4x}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h} = \lim_{h \to 0} 2x + h - 4 = 2x - 4$$

(8) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \frac{1}{1+x}$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{1+x+h} - \frac{1}{1+x}}{h} = \lim_{h \to 0} \frac{1}{h} \frac{1+x - (1+x+h)}{(1+x+h)(1+x)}$$

$$= \lim_{h \to 0} \frac{-1}{(1+x+h)(1+x)} = \frac{-1}{(1+x)^2}$$

(9) (10 points) find the derivative of $f(x) = 5x^{10} - 8e^x + 7$.

$$50x^9 - 8e^x$$

(10) (10 points) The graph of a function $f(x)$ is drawn below. Sketch the graph of $f'(x)$.

$y = f(x)$