Math 231 Calculus 1 Fall 13 Midterm 1a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-2, 3)$ and $(4, -1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

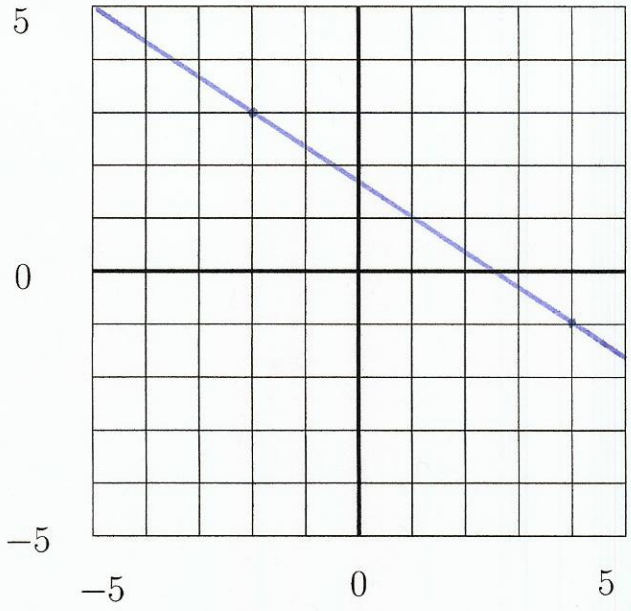


FIGURE 1

slope $= \frac{3-(-1)}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$

$y - 3 = -\frac{2}{3}(x+2)$

$y = -\frac{2}{3}x + 3 - \frac{4}{3}$

$y = -\frac{2}{3}x + \frac{5}{3}$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.

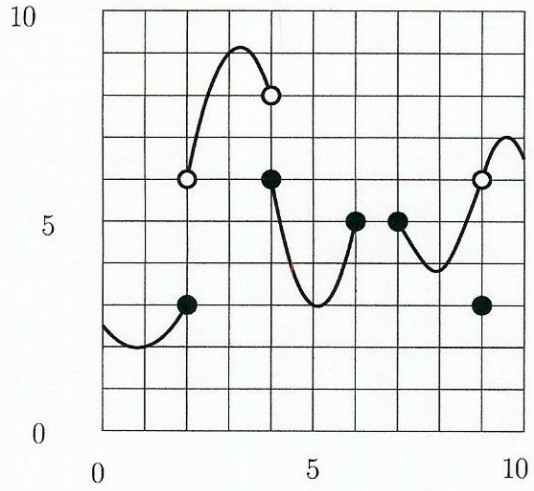


FIGURE 2. $f(x)$

(a) $\lim_{x\to 2-} f(x)$ 3

(b) $\lim_{x\to 4} f(x)$ DNE

(c) $\lim_{x\to 6-} f(x)$ 5

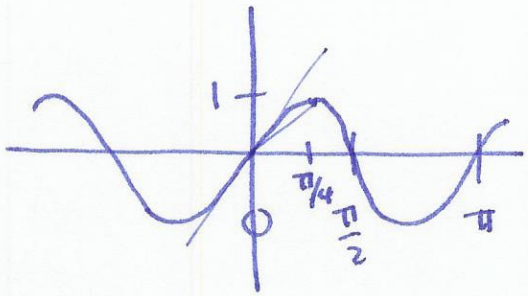
(d) $\lim_{x\to 6+} f(x)$ DNE

(e) $\lim_{x\to 9} f(x)$ 6

(3) (10 points) Sketch the graph of $f(x) = \sin 2x$.

(a) What is the average rate of change from $x = 0$ to $x = \pi/4$?

(b) Looking at the graph, do you expect this to be bigger or smaller than the actual rate of change at $x = 0$?



a) $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{1 - 0}{\pi/4 - 0} = \dfrac{4}{\pi}$

b) smaller

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 0} \frac{\sin 3x}{4x}$$

set $\theta = 3x \Leftrightarrow x = \theta/3$

$$\lim_{\theta \to 0} \frac{\sin \theta}{4\theta/3} = \frac{3}{4} \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \frac{3}{4}$$

(5) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

$$\lim_{x \to 3} \frac{(x-3)(x+2)}{(x-3)} = \lim_{x \to 3} x+2 = 5$$

(6) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{8}{x-4} - \frac{2}{\sqrt{x}-2}$$

$$\lim_{x\to 4} \frac{8 - 2(\sqrt{x}+2)}{(\sqrt{x}+2)(\sqrt{x}-2)} = \lim_{x\to 4} \frac{4-2\sqrt{x}}{(\sqrt{x}+2)(\sqrt{x}-2)} = \lim_{x\to 4} \frac{-2}{\sqrt{x}+2}$$

$$= -\frac{1}{2}$$

(7) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 3x$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - (x^2-3x)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h} = \lim_{h\to 0} 2x+h-3 = 2x-3$$

(8) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \frac{1}{1+x}$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{1+x+h} - \frac{1}{x+1}}{h} = \lim_{h\to 0} \frac{1}{h} \frac{x+1-1-x-h}{(1+x+h)(x+1)}$$

$$= \lim_{h\to 0} \frac{-1}{(1+x+h)(x+1)} = \frac{-1}{(1+x)^2}$$

(9) (10 points) find the derivative of $f(x) = 4x^{10} - 7e^x + 4$.

$f'(x) = 40x^9 - 7e^x$

(10) (10 points) The graph of a function $f(x)$ is drawn below. Sketch the graph of $f'(x)$.