Sample midterm 1 Solutions

Q1 a) 4 b) 6 c) 9 d) 5 e) 2 f) DNE

Q2 a) $\dfrac{4-4+12}{|-1|} = 12$

b) $\displaystyle\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5}\cdot\frac{\sqrt{x+4}+3}{\sqrt{x+4}+3} = \lim_{x\to 5} \frac{x+4-9}{(x-5)(\sqrt{x+4}+3)} = \lim_{x\to 5} \frac{1}{\sqrt{x+4}+3} = \frac{1}{6}$

c) $\displaystyle\lim_{x\to 0} \frac{\tan 4x}{3x} = \lim_{x\to 0} \frac{\sin 4x}{3x}\cdot\frac{1}{\cos 4x} = \underbrace{\lim_{x\to 0} \frac{4}{3}\frac{\sin 4x}{4x}}_{=4/3}\ \underbrace{\lim_{x\to 0}\frac{1}{\cos 4x}}_{=1}$

$= \dfrac{4}{3}.$

d) $\displaystyle\lim_{x\to 1+} \left(\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{x^2-1}}\right) = \lim_{x\to 1+} \frac{\sqrt{x+1}-1}{\sqrt{x^2-1}}\cdot\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$

$\displaystyle\lim_{x\to 1+} \frac{x+1-1}{\sqrt{x^2-1}(\sqrt{x+1}+1)} = \lim_{x\to 1+} \frac{x}{\sqrt{x^2-1}(\sqrt{x+1}+1)}$

$\displaystyle\lim_{x\to 1+} \frac{x}{\sqrt{x+1}+1} = \frac{1}{\sqrt{2}+1} \qquad \lim_{x\to 1+} \frac{1}{\sqrt{x^2-1}} = \infty$ so $\lim = \infty$.

Q3 $\displaystyle\lim_{x\to 2+} f(x) = \lim_{x\to 2+} \frac{\cos(\pi x)}{x} = \frac{1}{2}$

$\displaystyle\lim_{x\to 2+} f(x) = \lim_{x\to 2-} 2x + \frac{1}{2x-3} = 4+1 = 5$

} no value of c makes f cts.

Q4 $\dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \dfrac{4\pi(4)^2 - 4\pi(2)^2}{4-2} = 24\pi$

Q6 a) $\dfrac{(2x-2)-2x}{(2x-2)^2}$ b) $-6x^2e^x + -2x^3e^x$

c) $\dfrac{(x^2+4)\cdot 2x - (x^2-4)\cdot 2x}{(x^2+4)^2}$

Q7 a) $\dfrac{d}{dx}\left(\dfrac{-2}{(2x-2)^2}\right) = \cancel{\dfrac{d}{dx}\left(-2(2x-2)^{-2}\right)}\ \dfrac{(2x-2)^2\cdot 0 + 2(8x-8)}{(2x-2)^4}$

b) $-12xe^x - 6x^2e^x - 6x^2e^x - 2x^3e^x.$

c) $\dfrac{d}{dx}\left(\dfrac{16x}{(x^2+4)^2}\right) = \dfrac{(x^2+4)^2\cdot 16 - 16x(4x^3+16x)}{(x^2+4)^4}$

Q8 $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ $\qquad \lim\limits_{h\to 0} \dfrac{\dfrac{1}{\sqrt{2+h+2}} - \dfrac{1}{\sqrt{2+2}}}{h}$

$= \lim\limits_{h\to 0} \dfrac{1}{h}\left(\dfrac{1}{\sqrt{4+h}} - \dfrac{1}{2}\right) = \lim\limits_{h\to 0} \dfrac{1}{h}\ \dfrac{2-\sqrt{4+h}}{2\sqrt{4+h}}\ \dfrac{2+\sqrt{4+h}}{2+\sqrt{4+h}}$

$= \lim\limits_{h\to 0} \dfrac{1}{h}\ \dfrac{4-4-h}{2\sqrt{4+h}\,(2+\sqrt{4+h})} = \lim\limits_{h\to 0} \dfrac{-1}{2\sqrt{4+h}\,(2+\sqrt{4+h})} = \dfrac{-1}{4(2+2)} = \dfrac{-1}{16}$