Math 233 Calculus 3 Spring 12 Midterm 2b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

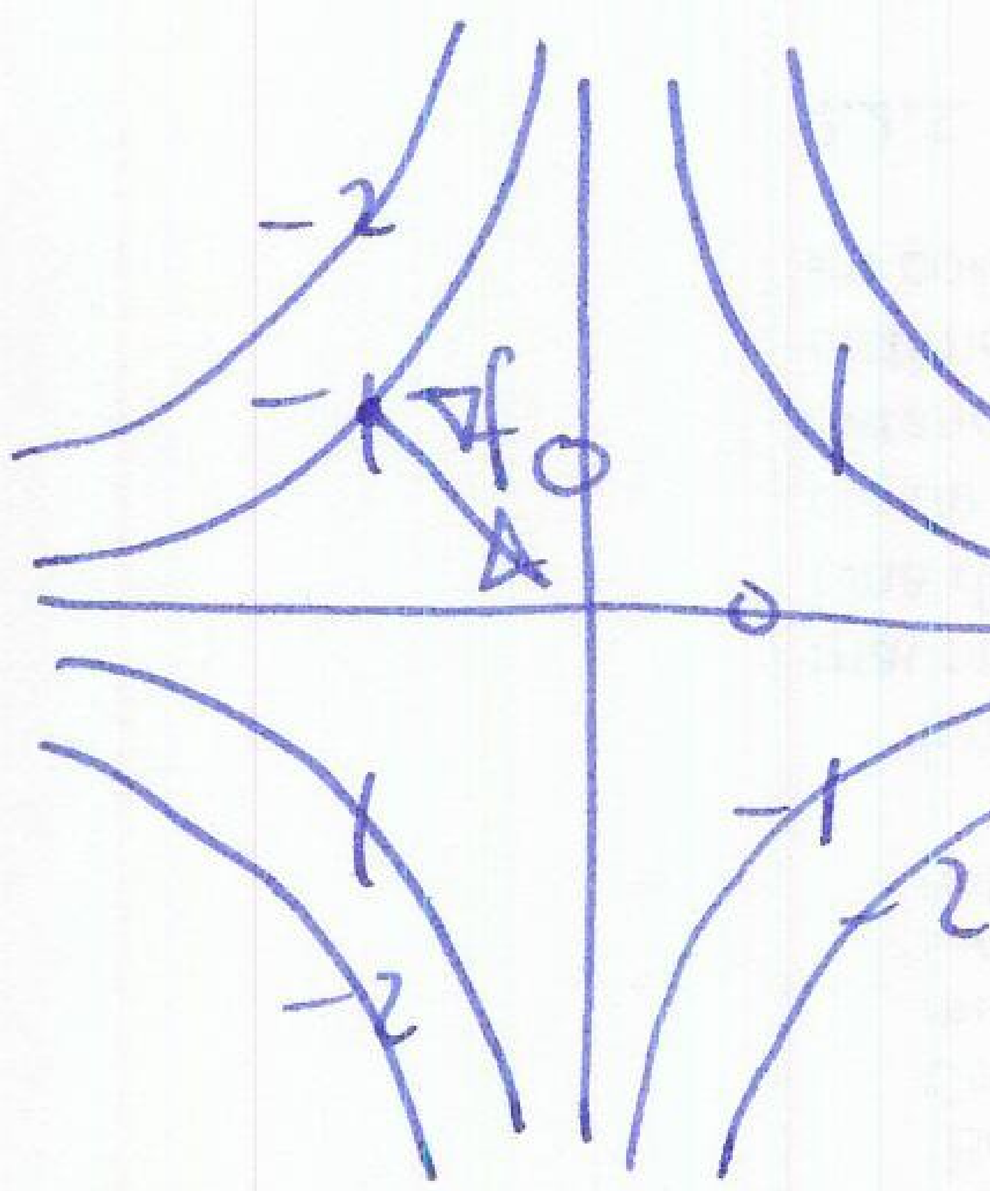
(1) (10 points)

(a) Sketch some level sets for the surface $z = xy$, and label them.

(b) Draw the gradient vector at the point $(-1, 1)$.

(c) Describe the surface.

$f(x,y) = xy$



$\nabla f = \langle y, x \rangle$

$\nabla f(-1,1) = \langle 1, -1 \rangle$

c) saddle surface

(2) (10 points) Show that the following limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{y^2 - x^2}{x^2 + y^2}$$

try $\displaystyle\lim_{(x,0)\to(0,0)} \frac{-x^2}{x^2} = -1$

$\displaystyle\lim_{(0,y)\to(0,0)} \frac{y^2}{y^2} = +1$

$\Rightarrow$ DNE

(3) (10 points) Find all first order partial derivatives of

$$f(x,y,z) = \frac{e^{-2xz}}{\cos(y+z)}.$$

$$f_x = \frac{e^{-2xz} \cdot -2z}{\cos(y+z)}$$

$$f_y = e^{-2xz} \cdot -(\cos(y+z))^{-2} \cdot (-\sin(y+z))$$

$$f_z = \frac{\cos(y+z) \cdot e^{-2xz} \cdot -2x - (-\sin(y+z)) \cdot e^{-2xz}}{\cos^2(y+z)}$$

(4) Find f_{yz} and f_{zz} if
$$f(x,y,z) = xe^{3yz} + \ln(y + xz).$$

$$f_y = xe^{3yz} \cdot 3z + \frac{1}{y+xz}$$

$$f_{yz} = xe^{3yz} \cdot 3 + xe^{3yz} \cdot 3y \cdot 3z + -(y+xz)^{-2} \cdot x$$

$$f_z = xe^{3yz} \cdot 3y + \frac{1}{y+xz} \cdot x$$

$$f_{zz} = xe^{3yz} \cdot 9y^2 + -(y+xz)^{-2} \cdot x^2$$

(5) (10 points) Find the equation of the tangent plane to the surface $z = xy + y^2$ at the point $(2, -1, \cancel{3})$. -1

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$f_x = y \qquad f_x(2,-1) = -1$

$f_y = x + 2y \qquad f_y(2,-1) = 0$

$$z = \cancel{3} - 1 - (x-2) + 0\cdot(y+1) = -1 - x + 2 = -x + 1$$

(6) (10 points) Find the normal vector to the surface $z = x^3 + xy - y^3$ at the point $(1, 1, -1)$.

Consider $f(x,y,z) = x^3 + xy - y^3 - z$

$\nabla f = \langle 3x^2 + y, x - 3y^2, -1 \rangle$

$\nabla f(1,1,-1) = \langle 4, -2, -1 \rangle = \underline{n}$

(7) (10 points) You are standing on the surface given by $z = y^2 - x$ at the point $(2, 2, 2)$. Which direction is the fastest way down?

$f(x,y) = y^2 - x \quad \nabla f = \langle -1, 2y \rangle$

$\nabla f(2,2) = \langle -1, 4 \rangle$

fastest way down $-\nabla f(2,2) = \langle 1, -4 \rangle$

(8) (10 points) Suppose you move on the path $r(t) = (t^2, t)$. Use the chain rule to find the rate of change along the path of $f(x, y) = x^2 + y^2$ at $t = 2$.

$r'(t) = \langle 2t, 1 \rangle \qquad \nabla f = \langle 2x, 2y \rangle$

$(f(r(t)))' = \nabla f(r(t)) \cdot r'(t)$

$= \langle 2t^2, 2t \rangle \cdot \langle 2t, 1 \rangle = 4t^3 + 2t$

at $t = 2 \qquad f(r(2))' = 4 \cdot 8 + 4 = 36$

(9) (10 points) Find the critical points of the function $f(x, y) = xy + y^2 - x$, i.e. the points where both f_x and f_y are zero.

$f_x = y - 1 = 0 \Rightarrow y = 1$

$f_y = x + 2y = 0 \Rightarrow x = -2$

critical point $(-2, 1)$

(10) (10 points) Find the linear linear approximation to $f(x, y, z) = xz - y$ at the point $(2, 1, -1)$.

$$L(x,y,z) = f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$$

$$\left.\begin{array}{l} f_x = z \\ f_y = -1 \\ f_z = x \end{array}\right\} \text{ at } (2,1,-1): \left\{\begin{array}{l} -1 \\ -1 \\ 2 \end{array}\right.$$

$$L(x,y,z) = 3 - (x-2) - (y-1) + 2(z+1)$$