Math 233 Calculus 3 Spring 12 Midterm 2a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points)

(a) Sketch some level sets for the surface $z = xy$, and label them.

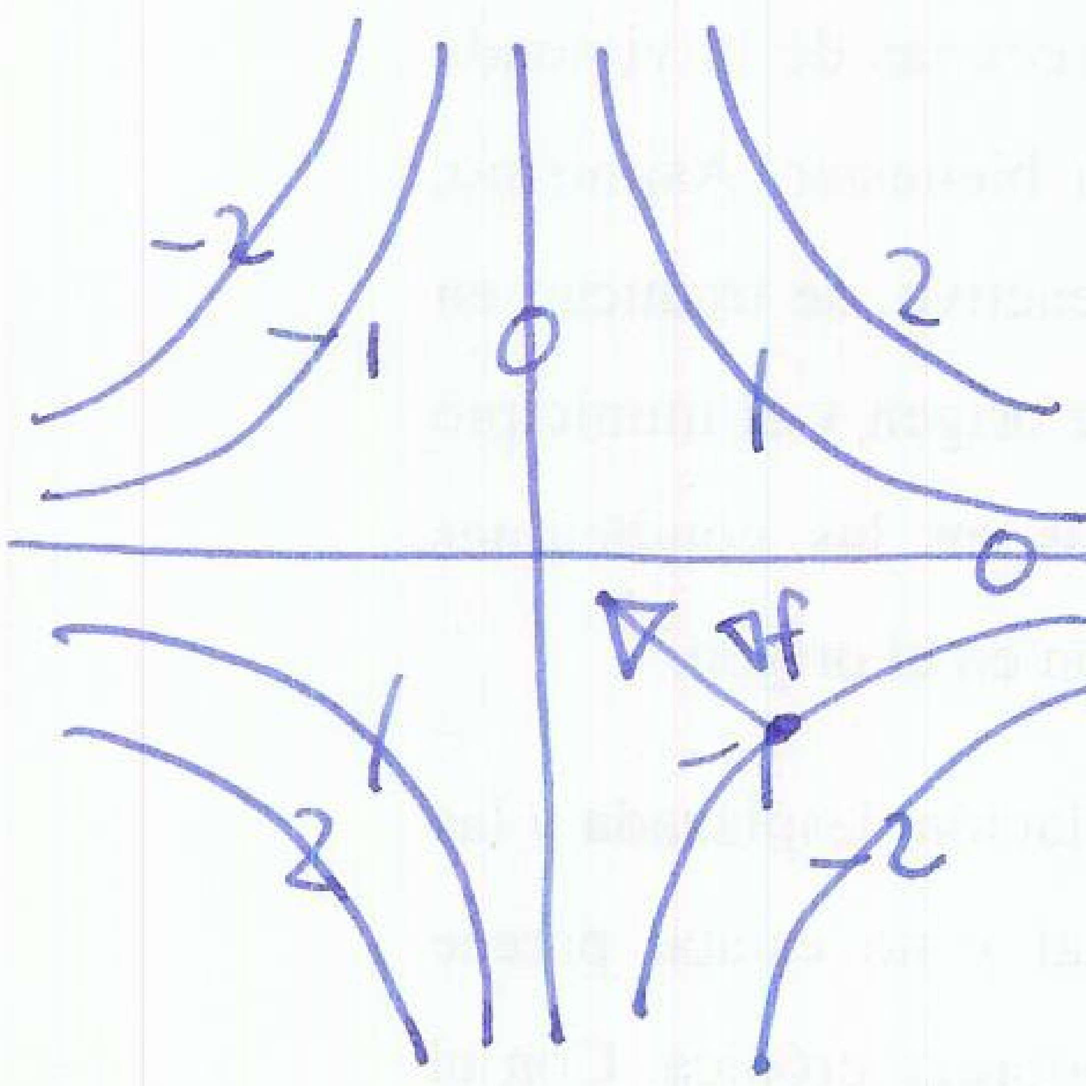
(b) Draw the gradient vector at the point $(1, -1)$.

(c) Describe the surface.

$f(x,y) = xy = c$

$\nabla f = \langle y, x \rangle$

$\nabla f(1,-1) = \langle -1, 1 \rangle$



c) saddle surface.

(2) (10 points) Show that the following limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

try:

$$\left.\begin{aligned} &\lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1 \\ &\lim_{(0,y)\to(0,0)} \frac{-y^2}{y^2} = -1 \end{aligned}\right\} \Rightarrow \text{DNE}$$

(3) (10 points) Find all first order partial derivatives of

$$f(x,y,z) = \frac{e^{-2xy}}{\cos(y+z)}.$$

$$f_x = \frac{-2y\, e^{-2xy}}{\cos(y+z)}$$

$$f_y = \frac{\cos(y+z) \cdot e^{-2xy} \cdot -2x - (-\sin(y+z)) \cdot e^{-2xy}}{\cos^2(y+z)}$$

$$f_z = e^{-2xy} \cdot -(\cos(y+z))^{-2} \cdot (-\sin(y+z))$$

(4) Find f_{xz} and f_{zz} if

$$f(x) = ye^{2xz} + \ln(x + yz).$$

$$f_x = ye^{2xt} \cdot 2z + \frac{1}{x+yz}$$

$$f_{xz} = ye^{2xz} \cdot 2 + ye^{2xz} \cdot 2x \cdot 2z + -(x+yz)^{-2} \cdot y$$

$$f_z = ye^{2xz} \cdot 2x + \frac{1}{x+yz} \cdot y$$

$$f_{zz} = ye^{2xz}\, 4x^2 + -(x+yz)^{-2} \cdot y^2$$

(5) (10 points) Find the equation of the tangent plane to the surface $z = xy + x^2$ at the point $(2, -1, 2)$.

$z = f(x,y)$ then tangent plane is $z = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$$\left.\begin{aligned} f_x &= y + 2x \\ f_y &= x \end{aligned}\right\} \text{at } (2,-1): \quad \begin{aligned} &3 \\ &2 \end{aligned}$$

so tangent plane is $z = 3(x-2) + 2(y+1)$

(6) (10 points) Find the normal vector to the surface $z = x^3 - xy - y^3$ at the point $(1, 1, -1)$.

consider $f(x,y,z) = x^3 - xy - y^3 - z$

$\nabla f = \langle 3x^2 - y, -x - 3y^2, -1 \rangle$

$\nabla f(1,1,-1) = \langle 2, -4, -1 \rangle = \underline{n}$.

(7) (10 points) You are standing on the surface given by $z = x^2 - y$ at the point $(2, 2, 2)$. Which direction is the fastest way down?

$\cancel{f(x,y,z) =}$ $f(x,y) = x^2 - y$

$\nabla f = \langle 2x, -1 \rangle$

at $(2,2)$: $\nabla f(2,2) = \langle 4, -1 \rangle$

fastest way down $= -\nabla f = \langle -4, 1 \rangle$.

(8) (10 points) Suppose you move on the path $r(t) = (t, t^2)$. Use the chain rule to find the rate of change along the path of $f(x, y) = x^2 + y^2$ at $t = 1$.

$r'(t) = (1, 2t)$ $\qquad$ $\nabla f = \langle 2x, 2y \rangle$

$(f(r(t)))' = \nabla f(r(t)) \cdot r'(t)$

$= \langle 2t, 2t^2 \rangle \cdot \langle 1, 2t \rangle$

$= 2t + 4t^3$

$t = 1 \; : \; (f(r(1)))' = 6$

(9) (10 points) Find the critical points of the function $f(x, y) = xy - y^2 + x$, i.e. the points where both f_x and f_y are zero.

$f_x = y + 1 = 0 \Rightarrow y = -1$

$f_y = x - 2y = 0 \Rightarrow x = -2$

critical point at $(-2, -1)$

(10) (10 points) Fidn the linear linear approximation to $f(x,y,z) = xy + z$ at the point $(3,1,-1)$.

$L(x,y,z) = f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$

$f_x = y$ at $(3,1,-1)$: 1

$f_y = x$ 3

$f_z = 1$ 1

$L(x,y,z) = \cancel{4} + (x-3) + 3(y-1) + (z+1)$

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