

## Math 233 Calculus 1 Spring 12 Sample midterm 2

- (1) Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy + x^2}{x^2 + y^2}$$

- (2) Find all the second order partial derivatives of

$$f(x, y, z) = x^2 \tan^{-1}(y + z)$$

- (3) Find the equation of the tangent plane to the surface  $z = x^2 - 4y^2$  at the point  $(1, 2, -15)$ .

- (4) Find the linear approximation to the function  $f(x, y, z) = e^{-2xy} + \cos(2yz)$  at the point  $(1, -1, 2)$ .

- (5) Find the normal vector to the surface  $z^2 = x^2 - y^2$  at the point  $(5, 3, 4)$ .

- (6) You are standing on a surface given by the equation  $z = x^2 - 2y^2$ . If you're standing at the point  $(2, 1, 0)$ , in which direction is the steepest slope?

- (7) The temperature in the solar system is given by

$$T(x, y, z) = \frac{10^5}{x^2 + y^2 + z^2}$$

If a comet travels along the path  $\mathbf{r}(t) = (t, t^2 - 4, 2t)$ , how fast is the temperature changing when  $t = 2$ ?

- (8) Find the critical points of the function  $f(x, y) = e^{4x} - e^{-y}$ . Use the second derivative test to classify them, if possible.

- (9) Find the extreme values of  $f(x, y) = 4x^2 + 3y^2$  on the square  $0 \leq x, y \leq 1$ .