MTH 233 Sample midterm 2 Solutions

Q1 try $x=0$: $\lim_{(0,y)\to(0,0)} \frac{0}{y^2} = 0$

try $y=0$: $\lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1$

$\Big\}$ different $\Rightarrow$ limit DNE

Q2 $f_x = 2x\tan^{-1}(y+z)$ $\quad f_y = \dfrac{x^2}{1+(y+z)^2}$ $\quad f_z = \dfrac{x^2}{1+(y+z)^2}$

$f_{xx} = 2\tan^{-1}(y+z)$ $\quad f_{yy} = -x^2\left(1+(y+z)^2\right)^{-2} . 2(y+z)$

$f_{zz} = -x^2\left(1+(y+z)^2\right)^{-2} 2(y+z)$ $\quad f_{xy} = f_{yx} = \dfrac{2x}{1+(y+z)^2}$

$f_{yz} = f_{zy} = -x^2\left(1+(y+z)^2\right)^{-2} . 2(y+z)$ $\quad f_{xz} = f_{zx} = \dfrac{2x}{1+(y+z)^2}$

Q3 $\frac{\partial z}{\partial x} = 2x$ $\quad \frac{\partial z}{\partial y} = -8y$

tangent plane:

$z = f(a,b) + \frac{\partial z}{\partial x}(a,b)(x-a) + \frac{\partial z}{\partial y}(y-b)$

$z = -15 + 2(x-1) + -16(y-2)$

Q4 $\frac{\partial f}{\partial x} = e^{-2xy} . -2y$ $\quad \frac{\partial f}{\partial y} = e^{-2xy} . -2x + -\sin(2yz) . 2z$

$\frac{\partial f}{\partial z} = -\sin(2yz) . 2y$ $\quad (a,b,c) = (1,-1,2)$

linear approx:

$f(x,y,z) = f(a,b,c) + \frac{\partial f}{\partial x}(a,b,c)(x-a) + \frac{\partial f}{\partial y}(a,b,c)(y-b) + \frac{\partial f}{\partial z}(a,b,c)(z-c)$

$e^{+2} + \cos(-4) + (2e^2)(x-1) + (-2e^2 - \sin(-2).4)(y+1) + (-\sin(-4).-2)(z-2)$

Q5 $\cancel{z = \sqrt{x^2+y^2} = (x^2+y^2)^{1/2}}$ consider $f(x,y,z) = x^2-y^2-z^2$

$\nabla f = \langle 2x, -2y, -2z\rangle$ $\quad \nabla f(5,3,4) = \langle 10, -6, -8\rangle$.

Q6 $f(x,y) = x^2 - 2y^2 \qquad \nabla f = \langle 2x, -4y \rangle$

$\nabla f(2,1,0) = \langle 4, -4 \rangle$

Q7 $(T(\underline{r}(t)))' = \nabla T(\underline{r}(t)) \cdot \underline{r}'(t)$

$\nabla T = 10^5 \langle -(x^2+y^2+z^2)^{-2} \cdot 2x, -(x^2+y^2+z^2)^{-2} \cdot 2y, -(x^2+y^2+z^2)^{-2} \cdot 2z \rangle$

$\underline{r}'(t) = \langle 1, 2t, 2 \rangle$

$t=2: \quad \underline{r}(2) = \langle 2, 0, 4 \rangle$

$\nabla T(\underline{r}(2)) = \nabla T(2,0,4) = \dfrac{10^5}{20^2} \langle 4,0,8 \rangle \cdot \langle 1,4,2 \rangle$

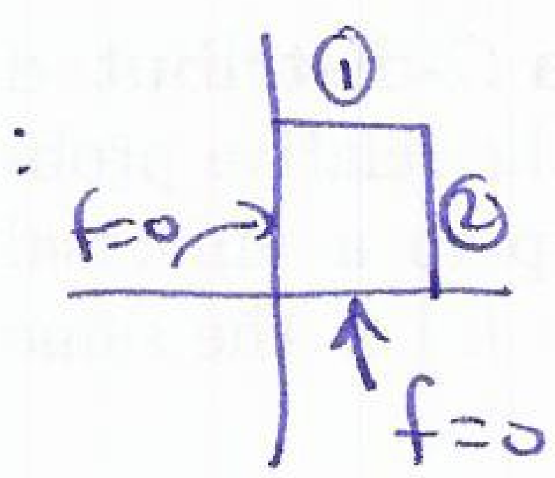
$= -\dfrac{10^5}{20^2} \cdot 20 = \dfrac{-10^5}{20} = -5 \times 10^3$.

Q8 $f(x,y) = e^{4x} - e^{-y} \qquad f_x = 4e^{4x} \qquad f_y = e^{-y}$ no critical points.

Q9 $f(x,y) = 4x^2 + 3y^2 \qquad f_x = 8x \qquad f_y = 6y$ critical point at $(0,0)$

$f(0,0) = 0$

boundary :



① $(x,1) \quad 0 \le x \le 1$

$f(x,1) = \cancel{4x+3y^2}\ 4x^2+3 \qquad f_x(x,1) = 8x$ critical point at $x=0$

so check endpoints $f(0,1) = 3$

$f(1,1) = 7$

② $(1,y) \quad 0 \le y \le 1$

$f(1,y) = 4 + 3y^2 \qquad f_y(1,y) = 6y$ critical point at 0

so check endpoints $f(1,0) = 4$

$f(1,1) = 7$

so global max = 7 global min = 0.