Math 233 Calculus 3 Spring 12 Midterm 1~~a~~ b

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find the angle between the two vectors $\langle 1, -2, 1\rangle$ and $\langle 1, -3, 1\rangle$.

$$\underline{v} \cdot \underline{w} = \|\underline{v}\| \, \|\underline{w}\| \cos\theta$$

$$\cos\theta = \frac{8}{\sqrt{6}\,\sqrt{11}} \qquad \theta = \cos^{-1}\left(\frac{8}{\sqrt{6}\sqrt{11}}\right.$$

A B

(2) (10 points) Find the area of the triangle with vertices $(1,-1,-1)$, $(1,3,-2)$ and $(2,1,3)$.

C

$\overrightarrow{AB} = \langle 0, 4, -1 \rangle$

$\overrightarrow{AC} = \langle 1, 2, 4 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -1 \\ 1 & 2 & 4 \end{vmatrix} = \langle 18, -1, -4 \rangle$$

area of triangle $= \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \| = \frac{1}{2}\sqrt{18^2+1+4^2} = \frac{1}{2}\sqrt{341}$

(3) (10 points) Find a parametric equation for the line of intersection of the two planes $x - y + z = 1$ and $x - y + 2z = 2$.

$\underline{n}_1 = \langle 1, -1, 1 \rangle$ $\underline{n}_2 = \langle 1, -1, 2 \rangle$

$$\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = \langle -1, -1, 0 \rangle$$

find point try $z=0$: $\left.\begin{array}{l} x-y=1 \\ x-y=2 \end{array}\right\}$ doesn't work

try $x=0$: $\left.\begin{array}{l} ① \ -y+z=1 \\ ② \ -y+2z=2 \end{array}\right\}$ ②−①: $z=1 \Rightarrow y=0$

so line is $\langle 0, 0, 1 \rangle + t \langle -1, -1, 0 \rangle$

(4) (10 points) Find the projection of the vector $\langle 4, -2, -1 \rangle$ onto the vector $\langle 2, -1, 1 \rangle$.

$\underline{u}$

$\underline{v}$

projection is $$\frac{\underline{u}\cdot\underline{v}}{\underline{v}\cdot\underline{v}}\,\underline{v} = \frac{8+2-1}{4+1+1} = \frac{9}{6}\langle 2,-1,1\rangle$$

(5) (10 points) Find the equation of the plane containing the line $\langle 0, 1, -3\rangle + t\langle 1, 1, 2\rangle$ and the point $(-1, 2, 1)$. (Hint: find the normal vector.)

plane contains $\langle 1,1,2\rangle$ and $\langle -1,1,4\rangle$

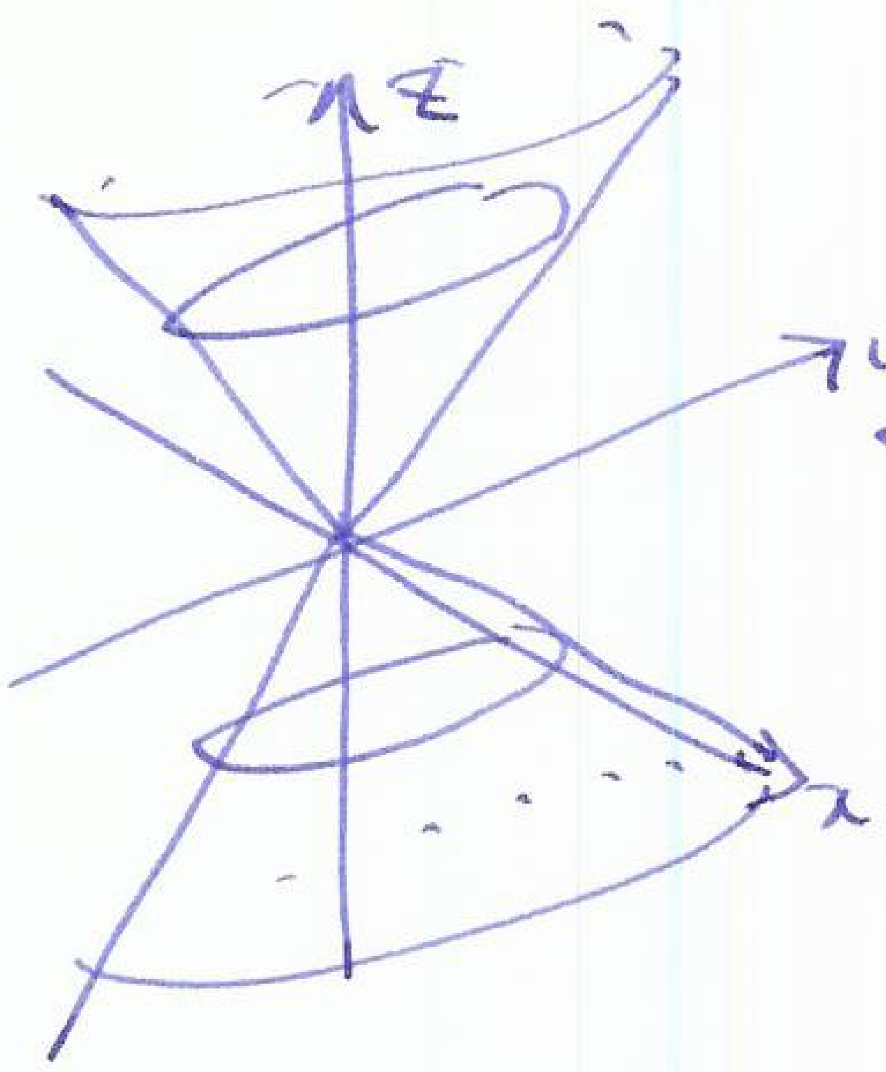
$$\underline{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 2 \\ -1 & 1 & 4 \end{vmatrix} = \langle 2, -6, 2\rangle$$

$$(\underline{x} - \langle 0,1,-3\rangle) \cdot \langle 2,-6,2\rangle = 0$$

$$2x - 6y + 2z = -12$$

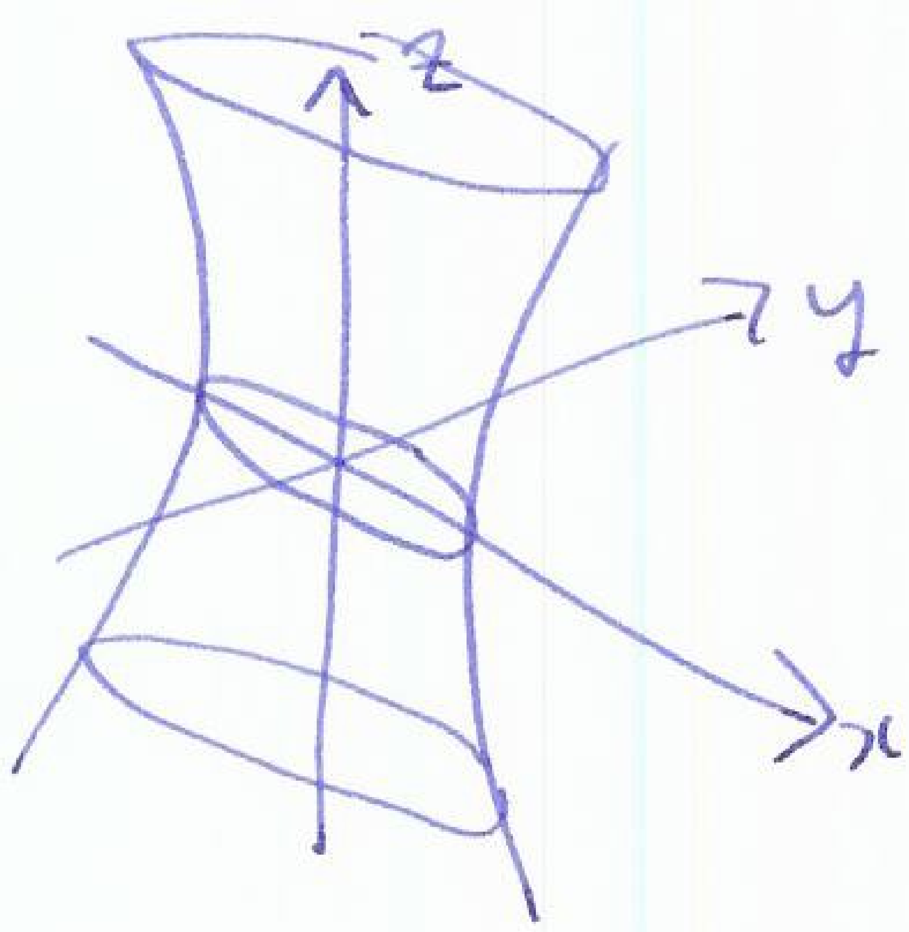
(6) (10 points) Sketch the surfaces and label their intersections with the coordinate axes.
(a) $z^2 = 4x^2 + y^2$
(b) $x^2 + 4y^2 = 9z^2 + 36$

a)



cone
intersects axes at $(0,0,0)$

b)



hyperboloid of one sheet
intersection with axes are:

$(\pm 6, 0, 0)$
$(0, \pm 3, 0)$

no intersections with z-axis

(7) (10 points) Write down a parameterization for the straight line segment from $(-1, 2, 1)$ to $(2, 4, 2)$. Use the integral formula for arc length to find the length of this line.

$\underline{r}(t) = \langle -1, 2, 1 \rangle + t \langle 3, 2, 1 \rangle \qquad 0 \leq t \leq 1$

$\| \underline{r}'(t) \| = \| \langle 3, 2, 1 \rangle \| = \sqrt{14}$

length $\int_0^1 \sqrt{14}\, dt = \left[\sqrt{14}\, t \right]_0^1 = \sqrt{14}$.

(8) (10 points) The position of a particle is given by $\mathbf{r}(t) = \langle \ln(t-1), e^{-t/2}, \tan(2x) \rangle$, find the acceleration of the particle.

$$\underline{r}'(t) = \left\langle \frac{1}{t-1}, -\frac{1}{2}e^{-t/2}, 2\sec^2(2x) \right\rangle$$

$$\underline{r}''(t) = \left\langle \frac{-1}{(t-1)^2}, \frac{1}{4}e^{-t/2}, 4\sec(2x) . \sec(2x)\tan(2x) . 2 \right\rangle$$

$$= \left\langle \frac{-1}{(t-1)^2}, \frac{1}{4}e^{-t/2}, 8\sec^2(2x)\tan(2x) \right\rangle$$

(9) (10 points) An object is thrown from the origin with initial velocity $\langle 20, 20, 20 \rangle$ m/s. Find an expression for the position of the object at time t it moves under the gravitational force $\mathbf{F} = \langle 0, 0, -gm \rangle$ m/s^2. Feel free to take $g = 10$.

$\underline{r}''(t) = \langle 0, 0, -10 \rangle$

$\underline{r}'(t) = \langle 0, 0, -10t \rangle + \underline{v}_0 \qquad \underline{v}_0 = \langle 20, 20, 20 \rangle$

$\underline{r}(t) = \langle 0, 0, -5t^2 \rangle + \underline{v}_0 t + \underline{r}_0 \qquad \underline{r}_0 = \langle 0, 0, 0 \rangle .$

(10) (10 points) The position of an object is given by $\mathbf{r}(t)$, and it moves with constant speed.
(a) Does the object have to move in a straight line?
(b) What can you say about $\mathbf{r}'(t)$?
(c) Show that $\mathbf{r}'(t)$ is perpendicular to $\mathbf{r}''(t)$.

a) no

b) $\|\underline{r}'(t)\| = c$

c) $\|\underline{r}'(t)\|^2 = c^2$

$\underline{r}'(t) \cdot \underline{r}'(t) = c^2$

diff wrt t :

$\underline{r}'(t) \cdot \underline{r}''(t) + \underline{r}''(t) \cdot \underline{r}'(t) = 0$

$2\underline{r}'(t) \cdot \underline{r}''(t) = 0$

$\Rightarrow \quad \underline{r}'(t) \cdot \underline{r}''(t) = 0$

so $\underline{r}'(t)$ and $\underline{r}''(t)$ are perpendicular.