Math 233 Calculus 3 Spring 12 Midterm 1a

Name: Solutions

- Do any 8 of the following 10 questions.
- You may use a calculator, but no notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find the nagle between the two vectors $\langle -1, 2, 1\rangle$ and $\langle 1, 1, -3\rangle$.

$\underline{v} \cdot \underline{w} = \|\underline{v}\| \, \|\underline{w}\| \cos\theta$

$-2 = \sqrt{6} \sqrt{11} \cos\theta \qquad \theta = \cos^{-1}\left(\frac{-2}{\sqrt{6}\sqrt{11}}\right)$

(2) (10 points) Find the area of the triangle with vertices $\overset{A}{(1,-1,1)}$, $\overset{B}{(2,3,4)}$ and $\underset{C}{(0,1,3)}$.

$\overrightarrow{AB} = \langle 1, 4, 3 \rangle$
$\overrightarrow{AC} = \langle -1, 2, 2 \rangle$

area of triangle = $\frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|$

$$\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 4 & 3 \\ -1 & 2 & 2 \end{vmatrix} = \langle 2, -5, 6 \rangle$$

area = $\frac{1}{2}\sqrt{4+25+36} = \frac{1}{2}\sqrt{75}$

(3) (10 points) Find a parametric equation for the line of intersection of the two planes $x + y - z = 1$ and $2x + y + z = 2$.

normal vectors $\underline{n}_1 = \langle 1, 1, -1 \rangle$

$\underline{n}_2 = \langle 2, 1, 1 \rangle$

$$\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & k \\ 1 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix} = \langle 2, -3, -1 \rangle$$ direction of line

point on line ~~try~~ $x = 0$ $\begin{rcases} (1)\ y - z = 1 \\ (2)\ y + z = 2 \end{rcases}$ $(1)+(2)$: $2y = 3$ $y = 3/2$

$\Rightarrow z = 1/2$.

so line is $\langle 0, \frac{3}{2}, \frac{1}{2} \rangle + t \langle 2, -3, -1 \rangle$

(4) (10 points) Find the projection of the vector $\underset{\text{u}}{\langle 4, -2, 1\rangle}$ onto the vector $\underset{\text{v}}{\langle 2, 3, -1\rangle}$.

projection is

$$\frac{\underline{u}.\underline{v}}{\underline{v}.\underline{v}}\,\underline{v} = \frac{8-6-1}{4+9+1}\langle 2,3,-1\rangle$$

$$= \frac{1}{14}\langle 2,3,-1\rangle$$

(5) (10 points) Find the equation of the plane containing the line $\langle 1, 0, -3\rangle + t\langle 1, 1, 1\rangle$ and the point $(4, 2, 1)$. (Hint: find the normal vector.)

plane contains $\langle 1,1,1\rangle$ and $\langle 3,2,4\rangle$

so $\underline{n} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 1 \\ 3 & 2 & 4 \end{vmatrix} = \langle 2,-1,-1\rangle$

equation of plane is $(\underline{x} - \langle 1,0,-3\rangle) \cdot \langle 2,-1,-1\rangle = 0$

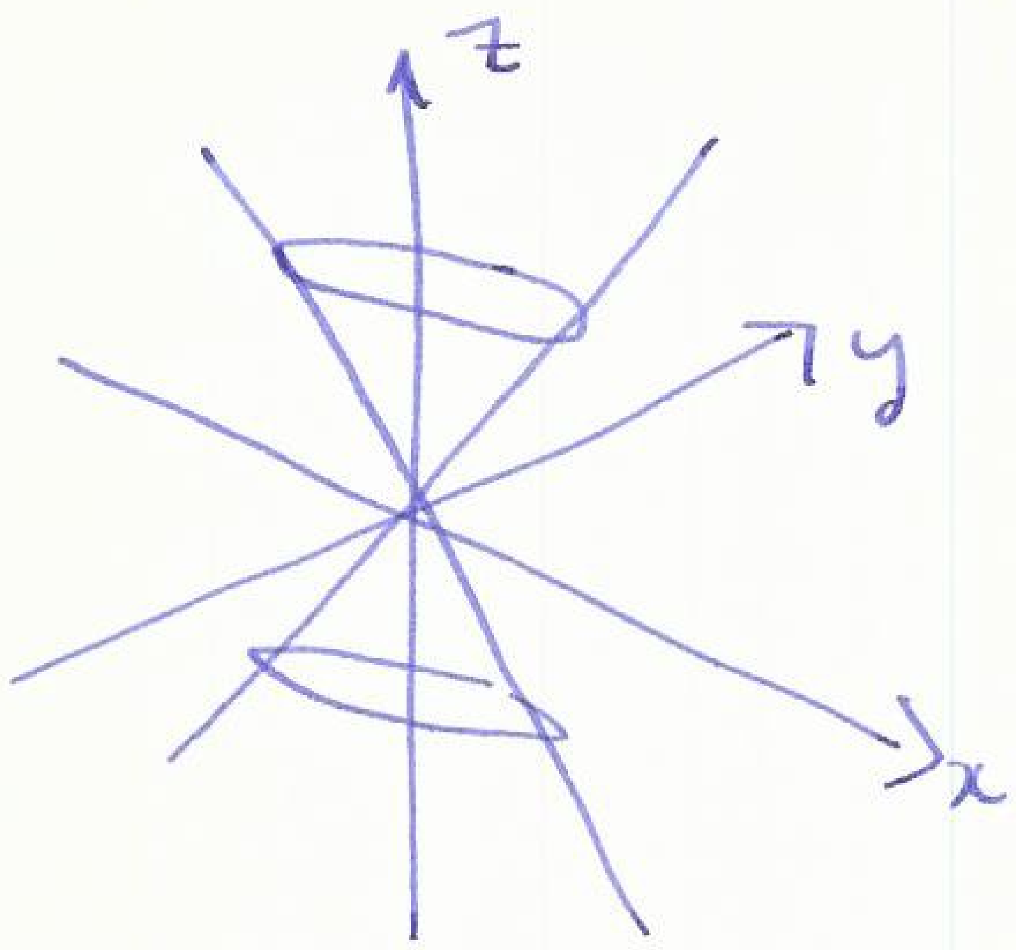
$2x - y - z = 5$

(6) (10 points) Sketch the surfaces and label their intersections with the coordinate axes.

(a) $z^2 = x^2 + 4y^2$

(b) $x^2 + 9y^2 = 4z^2 + 36$

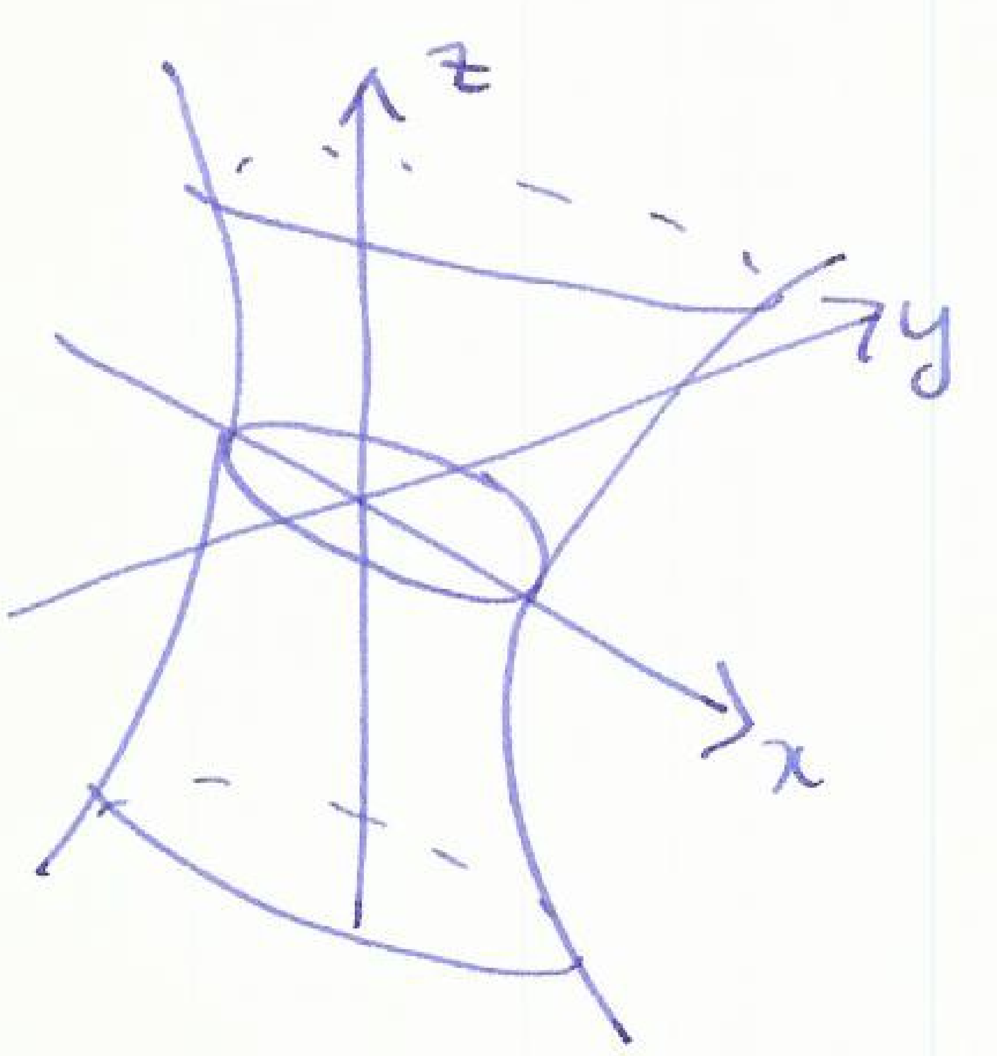
a)



cone

intersects axes at $(0,0,0)$ only

b)



one-sheeted hyperboloid

$(\pm 6, 0, 0)$

$(0, \pm 2, 0)$

& ~~not~~ no intersections with z-axis.

(7) (10 points) Write down a parameterization for the straight line segment from $(1, 2, -1)$ to $(2, 3, 5)$. Use the integral formula for arc length to find the length of this line.

$$\underline{r}(t) = \langle 1, 2, -1 \rangle + t\langle 1, 1, 6 \rangle \qquad 0 \leq t \leq 1$$

$$\|\underline{r}'(t)\| = \| \langle 1, 1, 6 \rangle \| = \sqrt{38}$$

$$\text{length} = \int_0^1 \sqrt{38}\, dt = \left[\sqrt{38}\, t\right]_0^1 = \sqrt{38}.$$

(8) (10 points) The position of a particle is given by $\mathbf{r}(t) = \langle \ln(t+1), e^{-2t}, \tan(x/2) \rangle$, find the acceleration of the particle.

$$\underline{r}'(t) = \left< \frac{1}{1+t}, \ -2e^{-2t}, \ \frac{1}{2}\sec^2\left(\frac{x}{2}\right) \right>$$

$$\underline{r}''(t) = \left< \frac{-1}{(1+t)^2}, \ 4e^{-2t}, \ \sec\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\cdot\frac{1}{2} \right>$$

$$= \left< \frac{-1}{(1+t)^2}, \ 4e^{-2t} \ \ \frac{1}{2}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right) \right>$$

(9) (10 points) An object is thrown from the origin with initial velocity $\langle 10, 10, 10\rangle$ m/s. Find an expression for the position of the object at time t it moves under the gravitational force $\mathbf{F} = \langle 0, 0, -gm\rangle$ m/s^2. Feel free to take $g = 10$.

$\underline{r}''(t) = \langle 0, 0, -10\rangle$

$\underline{r}'(t) = \langle 0, 0, -10t\rangle + \underline{v}_0 \qquad \underline{v}_0 = \langle 10, 10, 10\rangle$

$\underline{r}(t) = \langle 0, 0, -5t^2\rangle + \underline{v}_0 + \underline{r}_0 \qquad \underline{r}_0 = \langle 0, 0, 0\rangle.$

(10) (10 points) The position of an object is given by $\mathbf{r}(t)$, and it moves with constant speed.

(a) Does the object have to move in a straight line?

(b) What can you say about $\mathbf{r}'(t)$?

(c) Show that $\mathbf{r}'(t)$ is perpendicular to $\mathbf{r}''(t)$.

a) no

b) $\|\underline{r}'(t)\| = c$

c) $\|\underline{r}'(t)\|^2 = c^2$

$\|$

$\underline{r}'(t) \cdot r'(t)$ differentiate wrt t

$2\underline{r}'(t) \cdot \underline{r}''(t) = 0$

$\Rightarrow$ $\underline{r}'(t)$ perpendicular to $\underline{r}''(t)$.